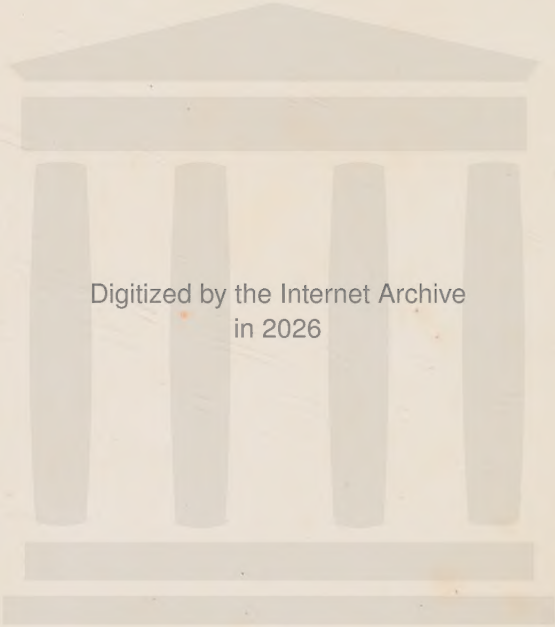
PRACTICAL ARITHMETIC
AND
MENSURATION

FRANK CASTLE

OW-IN-FURNESS



Digitized by the Internet Archive
in 2026

https://archive.org/details/practicalarithme0000fran

PRACTICAL ARITHMETIC AND MENSURATION

MACMILLAN AND CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
DALLAS · SAN FRANCISCO

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

PRACTICAL ARITHMETIC AND MENSURATION

BY

FRANK CASTLE, M.I.M.E.

MECHANICAL LABORATORY, ROYAL COLLEGE OF SCIENCE, SOUTH KENSINGTON
LATE LECTURER IN MATHEMATICS, ETC., AT THE MORLEY COLLEGE, LONDON

MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON

1912

COPYRIGHT.

First Edition 1908.
Reprinted 1909, 1912.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS
BY ROBERT MACLEHOSE AND CO. LTD.

PREFACE

An attempt has been made in the following pages to explain the ordinary processes of Arithmetic in a practical manner in accordance with modern ideas as to the needs of pupils attending evening continuation schools and technical classes. Conventional arrangements and the usual order of procedure have been discarded, and new rules are dealt with as they are required in the solution of practical problems. Moreover, as these rules are understood by the pupil, they are expressed in a general form, and algebraical ideas are in this way introduced at a very early stage.

As soon as the four simple rules have been revised, simple measurements with a centimetre scale form a natural explanation of decimal notation, and a foot-rule divided in the customary manner serves as an example of the need for understanding how to add and subtract vulgar fractions. The measurement of areas and volumes serve respectively as natural introductions to such processes as the multiplication and division of fractions, involution, evolution, and so on. In short, the plan has been always to make the pupil feel the need of a process before it is explained.

Similarly, the pupil is led to realise early the need for some labour-saving device to reduce the mechanical labour of computation; and logarithms are then introduced. Because of their practical importance, considerable space is devoted to the explanation of logarithms, and a simple method by which

they may be calculated is given. When the pupil may be presumed to understand them, logarithms are used in the solution of every example where this procedure reduces the labour involved. Consistent encouragement is given, therefore, to the solution by logarithms of all complicated problems such as those on interest and the mensuration of certain figures and solids. The result is that at least half the problems in the book are examples of the use of logarithms in calculation. Since the movement for simplified and practical methods of mathematical teaching was commenced by Professor J. Perry, F.R.S., several years ago, work with logarithms, the slide-rule, and other aids to calculation has extended greatly in schools. The present volume partakes of the spirit of these reforms by the prominence given in it to actual problems and practical methods of dealing with them.

The order of treatment in the book is that which the author has found effective in teaching students of average ability; but there is no reason why the sections should not be taught in any order the teacher considers better.

I find it difficult adequately to express how much I am indebted to Professor R. A. Gregory and to Mr. A. T. Simmons for again placing their extensive experience at my disposal; both in the revision of the manuscript and in the passage of the book through the press they have been of continual assistance. Any merit the volume may have is due largely to their able help.

FRANK CASTLE.

LONDON, *July* 1908.

CONTENTS

PAGE

CHAPTER I

The Four Simple Rules. Generalised Arithmetic, Brackets. Measurement of Length (British). Decimals, Addition and Subtraction. Metric Measurement. Squared Paper. Vulgar Fractions, Addition and Subtraction 1

CHAPTER II

Measurement of Angles. Construction of Plane Figures. Measurement of Area. Area of Rectangle. Vulgar Fractions: Multiplication, Division. Conversions and Simplifications. Metric Measures of Area. Decimals: Multiplication, Contracted Methods; Division, Contracted Methods. Decimals of Concrete Quantities. Averages . . . 29

CHAPTER III

Measurement of Volume. Volumes of Liquids. Involution. Evolution 62

CHAPTER IV

Mass and Weight. Ratio. Specific Gravity. Proportion. Variation. Percentages. Interest 75

CHAPTER V

Indices. Logarithms. Multiplication and Division by Logarithms. Involution and Evolution by Logarithms. Calculation of Logarithms. Simple and Compound Interest. Discount. Present Worth. Stocks and Shares. . . . 97

PAGE

CHAPTER VI

Area of Rectangle, Parallelogram; Practical Applications. Addition and Subtraction of Areas. Area of a Triangle, Equilateral Triangle, Hexagon, Trapezium, Quadrilateral . 126

CHAPTER VII

Circumference of Circle. Circumference of Ellipse. Arches. Area of Circle, Ellipse. Area of Irregular Figure. Simpson's Rule. Slide Rule. Surface of Prism, Cylinder, Pyramid, Cone, Sphere 140

CHAPTER VIII

Measurement of Volume; Prism, Cube, Cylinder, Hollow Cylinder 169

CHAPTER IX

Volume of Pyramid. Frusta of Pyramid and Cone. Volume of Sphere. Similar Solids 180

CHAPTER X

Angular Measurement. Ratios of an Angle; Sine, Cosine, Tangent. Use of Tables 195

CHAPTER XI

Arithmetical Evaluation 209

TABLES 224

ANSWERS 233

INDEX 247

CHAPTER I.

The Four Simple Rules. Generalised Arithmetic, Brackets. Measurement of Length (British). Decimals, Addition and Subtraction. Metric Measurement. Squared Paper. Vulgar Fractions, Addition and Subtraction.

THE FOUR SIMPLE RULES.

Addition.—The **sum** of two or more numbers is a single number obtained by adding the numbers together.

The process of finding the sum is called **addition** of the numbers.

The sign used to indicate addition is +, called plus.

Thus $5 + 4 = 9$.

The sign = is read as "is equal to" or "equals." Thus, 5 added to 4 equals 9.

EXAMPLE. *Find the sum of*

(i)

6187
456
58
6701

(ii)

£	s.	d.
278 .	5 .	$4\frac{1}{2}$
37 .	4 .	$11\frac{3}{4}$
3784 .	15 .	$4\frac{3}{4}$
4100 .	5 .	9

In (i), by adding the numbers on the right, or units, column, the sum 21 is obtained. This is 2 tens and 1 unit. The one unit is put in the units column and the two tens are carried to the next, or tens, column. The sum is now 18 tens, and with the 2 carried from the last column gives 20 tens or 200. The 0 is entered and the two hundreds are carried to the next, or hundreds, column, giving 7. Finally, writing down the 6 from the thousands column the result is 6701.

In (ii), the addition of the numbers in the pence column gives 21*d.* or 1*s.* 9*d.*; the 9 is written in the pence column and the 1 is carried to the shillings column, which becomes 25*s.* or £1. 5*s.* The 5 is put in the shillings column and the 1 carried to the next, or pounds, column, the sum of the pounds being 4100.

EXERCISES I.

Find the sum of

1.	2345 6982	**2.**	3547 7109	**3.**	4915 6394
4.	45263 76589 6108 123 67	**5.**	79623 54132 3475 672 65	**6.**	658973 784 9635 6978 25

Find the sum of

7. 24315, 80164, 35827 and 42911.

8. 3751, 76108, 53, 29857, 341598.

9. 29156, 523144, 646, 51, 7909, 521584.

10. 40628, 925814, 926851, 593371, 3910, 460.

11. £259. 19*s.* 3*d.*, £192. 0*s.* 9*d.*, £207. 11*s.* 6*d.*

12. £824. 14*s.* 8*d.*, £51. 10*s.* 11*d.*

13. £876. 5*s.* 7*d.*, £167. 14*s.* 0*d.*

14. £377. 10*s.* 0*d.*, £264. 5*s.* 0*d.*, £22. 13*s.* 8*d.*

15. £37. 17*s.* 4½*d.*, £19. 12*s.* 10*d.*, £183. 19*s.* 4½*d.*, and £52. 0*s.* 11¾*d.*

Multiplication.—The process of addition is used to find the sum of several different numbers. When the same number is repeated several times, another process called **multiplication** is employed to find their sum.

The sign of multiplication is $\times$.

Thus, $4 \times 3 = 12$.

EXAMPLE 1. *Multiply* 376; (i) *by* 3, (ii) *by* 36.

(i)	(ii)	
376	376	376
3	36	3
1128	2256	1128
	1128	12
	13536	13536

In (i), we say 3×6 makes 18; we write down the 8 and carry the one ten. Next $3 \times 7 + 1$ is 22; set down 2 and carry 2 to the next figure. Then $3 \times 3 + 2 = 11$, and the result is 1128.

In (ii), the product may be obtained either by multiplying 376 by 36 or by its **factors** 3 and 12.

The latter form is usually convenient in questions dealing with money, weight, etc.

EXAMPLE 2. *Multiply* £1. 5*s*. 6½*d*. ; (i) *by* 9, (ii) *by* 48.

(i)	£	*s*.	*d*.
	1	5	6½
			9
	11	9	10½

(ii)	£	*s*.	*d*.
	1	5	6½
			4
	5	2	2
			12
	61	6	0

As $9 \times \frac{1}{2}d. = 4\frac{1}{2}d.$ the ½*d*. is written down and 4*d*. carried to the pence column. Next $9 \times 6 + 4 = 58d. = 4s.\ 10d.$; the 10*d*. is set in the pence column and the 4*s*. carried to the next column ;

$$9 \times 5 + 4 = 49s. = £2.\ 9s.$$

The 9*s*. is placed in the shillings column and the £2 carried. Finally, $9 \times 1 + 2 = £11$. The result is £11. 9*s*. 10½*d*.

EXERCISES II.

Tables of Money, Weights and Measures will be found on p. 224.

Multiply

1. 74536 by 8.
2. 725382 by 12.
3. 948061 by 308.
4. 52473 by 640.
5. 9358 by 7008.
6. 67849 by 736.
7. 6½*d*. by 4 and 24.
8. 2*s*. 9¾*d*. by 5 and 11.
9. £27. 11*s*. 7½*d*. by 14.
10. 13 cwt. 11 lbs. 7 oz. by 16.
11. 1 ton 7 cwt. 6 lb. by 26.
12. £24. 14*s*. 7½*d*. by 68.
13. 37 yds. 2 ft. 8 in. by 315.
14. £6. 5*s*. by 0·001.

Subtraction.—The process by which the **difference** of two numbers is obtained is called **subtraction**. The difference between the two numbers added to the smaller number will make it equal to the larger number.

The sign of subtraction is −, read as **minus.** Thus, $7 - 3 = 4$ is read as 7 minus 3 equals 4.

EXAMPLE 1. *Subtract* 7465 *from* 9378.

EXAMPLE 2. *Find the difference between* £29. 17*s*. 7*d*. *and* £15. 13*s*. 4*d*.

(1)	
	9378
	7465
	1913

(2)	£	*s*.	*d*.
	29	17	7
	15	13	4
	14	4	3

(i) Taking 5 units from 8 units, the remainder 3 units is set down in the units column. Similarly, 6 tens from 7 tens give 1 ten, which is written in the tens column. Next, as 4 hundreds cannot be taken from 3 hundreds, it is necessary to take one of the thousands or ten hundreds

and to subtract 4 hundreds from 13 hundreds, obtaining 9, which is set down in the hundreds column. Finally, 1 thousand from 9 thousands leaves 8 thousands, and subtracting 7 thousands 1 thousand remains, and this is set down in the thousands column. The preceding process is shortened in practice : 5 from 8 is 3, 6 from 7 is 1, 4 from 13 is 9, and finally, 8 from 9 is 1. The result obtained may be verified by adding 1913 to 7465 ; the sum is 9378. This is called **Complementary Addition.**

In (ii), commence by taking 4*d.* from 7*d.* and set down the remainder 3*d.* in the pence column. Next 13*s.* from 17*s.* gives 4*s.*, and finally, £15 from £29 gives £14. As before, the sum obtained by adding the two last lines should be equal to the upper line.

EXERCISES III.

Perform the following subtractions :

1. 45461 38947	**2.** 725624 469732	**3.** 574324 398763
4. 914572 356793	**5.** 573124 490567	**6.** 79601 649

Find the difference between

7. 987948 and 735831. **8.** 674531 and 46217.

9. £43. 9*s.* 7*d.* and £63. 14*s.* 6*d.* **10.** £122. 8*s.* 10*d.* and £151. 6*s.* 8*d.*

11. 3 tons 7 cwt. and 12 tons 5 cwt.

Division.—The symbol used to denote division may be $\div$, :, or simply a horizontal line. Thus, 8 divided by 2 may be written as $8 \div 2$, or $8 : 2$, or $\frac{8}{2}$.

The correct result, 4, can be found also by repeated subtraction, thus $8 - 2 = 6$, $6 - 2 = 4$.

In $8 \div 2 = 4$, 8 is called the **dividend**, 2 the **divisor** and 4 the **quotient.**

Example 1. *Divide* 32304 *by* 48.

The given number may be divided either by 48 or by its factors 6 and 8 or 12 and 4, since $6 \times 8 = 48$ and $12 \times 4 = 48$.

The process of dividing by the factors of a number is especially useful in questions involving money, weights, lengths and so on.

```
48 ) 32304 ( 673
     288
     ---
      350
      336
      ---
       144
       144
       ---
```

```
8 | 32304
6 | 4038
    673
```

EXAMPLE 2. *Divide* 13*s.* 6*d.* *by* 6.

	s.	*d.*
6	13	6
	2	3

In (i), as 32304 is greater than 600×48 and less than 700×48, subtract $600 \times 48 = 28800$ from 32304; or, as shown, merely write the first three figures 288 under 323; subtract and the remainder 3504 is obtained. This is greater than 70×48 and less than 80×48; hence subtract 70×48 or 3360 from 3504, or better, 336 from 350; and as the remainder 144 is 3×48, the quotient is 673.

To verify the result, test if *Quotient* $\times$ *Divisor* + *Remainder* = *Dividend*,

i.e. $673 \times 48 \div 0 = 32304$.

In (ii), 13*s.* divided by 6 gives 2*s.* and 1*s.* over; as $1s. = 12d.$ and $12 + 6 = 18$, then $18 \div 6 = 3d.$

To verify, $6 \times (2s.\ 3d.) = 13s.\ 6d.$

EXAMPLE 3. *Find the number of times that* 4*d.* *is contained in* £3. 6*s.* 8*d.*

In cases of this kind it is necessary to reduce both the given expressions to the *same denomination*, and finally to divide one by the other.

Thus £3. 6*s.* 8*d.* is reduced to pence. Multiplying £3 by 20, the number of shillings in a pound, $3 \times 20 = 60s.$ and £3. $6s. = 66s.$

Again, multiplying by 12, the number of pence in a shilling, then $66 \times 12 + 8 = 800d.$

Hence £3. 6*s.* $8d. = 800d.$

$\therefore\ 800 \div 4 = 200.$

Hence 4*d.* is contained 200 times in £3. 6*s.* 8*d.*

EXERCISES IV.

Divide

1. 39548 by 4.
2. 253809 by 9.
3. 2783550 by 70.
4. 743235 by 5.

In the following find both quotient and remainder:

5. $374807 \div 56$.
6. $59492 \div 13$.
7. $969234 \div 17$.
8. $17324 \div 480$.

Find, by reducing each quantity to its least common term, the number of times that

9. 6*d.* is contained in £5. 7*s.* 6*d.*
10. 2*s.* 6*d.* is contained in £71. 7*s.* 6*d.*
11. 19*s.* 2*d.* is contained in £77. 12*s.* 6*d.*

12. Find, by reducing each quantity to its least common term, the number of times that 5*s.* 4*d.* is contained in £5. 1*s.* 4*d.*

Divide

13. £53. 8*s.* 9*d.* by 540.

14. (2 days 17 hours 30 min. 6 sec.) ÷ 6.

15. (425 tons 15 cwt. 2 qrs. 12 lbs.) ÷ 27.

Generalised Arithmetic.—Arithmetical processes may be expressed in a *general* manner by using letters in addition to numbers to indicate various quantities.

Thus, just as 6*d.* (sixpence) means a sum of six pennies, so $6a$ means six times the quantity a, whatever a may be. If a be 4, then $6a$ will denote $6 \times 4 = 24$.

The number, such as 6, prefixed to one or more given letters is called a **coefficient**. The product of two quantities is usually written with no sign between them, *e.g.* $6 \times a$ is written $6a$. When the coefficient is 1 it is not written: thus $1 \times a$ is written a.

Like quantities are those which differ only in their numerical coefficients. Thus $3a$, $6a$, $4a$, $-9a$ are like quantities.

Generalised Addition.—The addition of like quantities simply consists in finding the sum of the coefficients and writing after this sum the common letter, or letters.

EXAMPLE 1. *Find the sum of* $3a$, $6a$ *and* $4a$.

$$3a + 6a + 4a = 13a.$$

This expression is true for all values of a; and this fact may be verified by substituting in the expression any special value for a.

Thus, if $a = 2$, then $3a = 6$, $6a = 12$ and $4a = 8$, and

$$3a + 6a + 4a = 6 + 12 + 8 = 26.$$

The process of verification is the same when two or more letters occur in an expression.

EXAMPLE 2. *Find the sum of* $3abc$, $6abc$, $4abc$.

$$3abc + 6abc + 4abc = 13abc.$$

This sum may be verified as in Example 1 by substituting values for a, b and c.

The sum of a series of different quantities, or **terms** as they are called, may be obtained by placing like quantities under

each other in vertical columns, adding the + or positive terms, and then subtracting from the result the - or negative terms.

EXAMPLE 3. *Add together* $4ab$, $6ab$, $5ab$, $-3ab$, $2ab$. Verify the result when $a=2$, $b=1$.

$4ab$	8
$6ab$	12
$5ab$	10
$-3ab$	-6
$2ab$	4
$14ab$	28

The sum of the coefficients is 14, and by adding the common letters a, b we obtain the result $14ab$.

When $a=2$ and $b=1$, $14ab$ is equal to 28.

The Negative Sign.—To explain the use of the $-$, or *negative* sign as it is called, in the example just worked, it may be mentioned that although in Arithmetic we cannot take 9 from 6, we may, by the use of the negative sign, express the result of such a process and write that result as -3.

For example, a thermometer reading 6° C. denotes 6 degrees above the freezing point on a Centigrade thermometer. If, now, the temperature is reduced 9 degrees, the new reading of the thermometer may be referred to either as 3 degrees below the freezing point or, better, as $-3°$ C.

Again, if a man holds £6 but owes £9, his wealth may be designated as $-$£3; or, in other words, he is said to be £3 in debt.

EXAMPLE 4. *Add together* $3a+4b+5c$, $4a-3b-2c$, $-6a+2b+7c$. *Verify the result when* $a=2$, $b=3$, $c=4$.

$3a+4b+5c$	$(3\times2)+(4\times3)+(5\times4)=$	38
$4a-3b-2c$	$(4\times2)-(3\times3)-(2\times4)=$	-9
$-6a+2b+7c$	$-(6\times2)+(2\times3)+(7\times4)=$	22
$a+3b+10c$		51

Arrange the terms as shown; then $7c+5c=12c$ and $12c-2c=10c$.

Similarly, $2b+4b-3b=3b$ and $3a+4a-6a=a$.

When $a=2$, $b=3$ and $c=4$, the result is $2+(3\times3)+(10\times4)=51$.

The symbols ( ), called **brackets**, are used to indicate that the product of the two numbers in the first bracket must be

added to the product of the numbers in the next bracket, and finally the sum so obtained added to the product of the numbers in the remaining bracket.

Generalised Multiplication.—The processes of multiplication when letters and numbers occur in the various terms are similar to those adopted in the case of numbers.

The product a and a may be written $a \times a$, but is usually expressed by placing a small number to the right, and near the top of the quantity, thus $a \times a$ is written a^2, and is read as a to the **second power** or a **squared**: $a \times a \times a$ is written a^3, and is read as a to the **third power** or a **cubed**: similarly, $a \times a \times a \times a$ is written a^4, and is read as a to the **fourth power**.

EXAMPLE 1. *Find the product of* $3a$ *and* $7a$. *Verify the result when* $a=2$.

The product of 3 and 7 is 21.

Hence, $3a \times 7a = 21a^2$.

To verify, $3a = 3 \times 2 = 6$ and $7a = 14$;

$\therefore\ 3a \times 7a = 6 \times 14 = 84$.

Also $21a^2$ or $21a \times a = 21 \times 2 \times 2 = 84$.

EXAMPLE 2. *Multiply* $4a^2$ *by* $5a^3$.

The product of 4 and 5 is 20.

Also, as $a^2 = a \times a$ and $a^3 = a \times a \times a$,

$a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5$.

The number 5 means that a is multiplied by itself 5 times:

$\therefore\ 4a^2 \times 5a^3 = 20a^5$.

EXAMPLE 3. $3ab \times 8ab^2$.

Here $3 \times 8 = 24$ and $ab \times ab^2 = a^2 \times b^3$; $\therefore\ 3ab \times 8ab^2 = 24a^2b^3$.

EXAMPLE 4. $(x+5)(x+6)$.

$$\begin{array}{r} x+5 \\ x+6 \\ \hline x^2+5x \\ \quad 6x+30 \\ \hline x^2+11x+30 \end{array}$$

Arrange the terms as in ordinary multiplication; the product of x and $x+5$ is x^2+5x. Next multiply $x+5$ by 6; the product is $6x+30$. Place this result one column to the right, so that like terms $5x$ and $6x$ are in the same vertical column. Add, and the sum is found to be $x^2+11x+30$.

Rule of Signs.—$7a \times 3$ means that $7a$ has to be *added* 3 times;

$$\therefore\ 7a \times 3 = 7a + 7a + 7a = 21a.$$

Similarly, $a \times b$ indicates that a has to be added b times;

$$\therefore\ a \times b = a + a + a \ldots \text{ to } b \text{ terms} = ab.$$

$7a \times (-3)$ means that $7a$ has to be *subtracted* 3 times;

$$\therefore\ 7a \times (-3) = -7a - 7a - 7a = -21a,$$

and $a \times (-b) = -a - a - a \ldots$ to b terms $= -ab$.

Proceeding in this manner we can understand what is called the **Rule of Signs**, viz. the product of *like* signs gives *plus*, of *unlike* signs gives *minus*. For example,

$$(-7a) \times (-3) = 21a \quad \text{and} \quad (-a) \times (-b) = ab.$$

EXAMPLE 5. *Find the product of* $x - 5$ *and* $x + 6$.

$$\begin{array}{r} x - 5 \\ x + 6 \\ \hline x^2 - 5x \\ \quad 6x - 30 \\ \hline x^2 + \ x - 30 \end{array}$$

Multiply $x - 5$ by x and obtain $x^2 - 5x$. Notice that the unlike signs x and -5 give $-5x$. Next multiply by 6; the product is $6x - 30$. Now add the two results and $x^2 + x - 30$ is obtained.

In this result, as in that of Ex. 4, the product consists of three terms; the first term is the product of the two first terms of the factors, the coefficient of x in the second term is the sum of the second terms of the factors (thus $6 - 5 = 1$), and the third term is their product.

Generalised Subtraction.—The process of subtraction when letters occur in a given expression is in many respects similar to the arithmetical method.

EXAMPLE 1. *Subtract* $3d$ *from* $7d$.

Subtract 3 from 7 and insert the common letter d, the result is $4d$. Similarly, $7a - 3a = 4a$.

Set off a length AB of 7 units, the distance between each being a; then 3 units measured in the opposite direction

from B to C determines a length $AC = 4a$. Thus, to subtract one term from another it is only necessary to change the

FIG. 1.

sign of the term to be subtracted and add. For example, the sum of $7a - (3a) = 4a$.

EXAMPLE 2. *Subtract* $3a + 3b - 4c$ *from* $7a - 5b - 8c$, *and verify the result when* $a = 3$, $b = 5$, $c = 4$.

Arrange the terms under each other:

$7a + 5b + 8c$	$(7 \times 3) + (5 \times 5) + (8 \times 4) = 78$
$3a + 3b + 4c$	$(3 \times 3) + (3 \times 5) + (4 \times 4) = 40$
$4a + 2b + 4c$	38

$$(4 \times 3) + (2 \times 5) + (4 \times 4) = 38.$$

Mentally change the signs of the terms in the lower line: then the sum of $+8c$ and $-4c$ is $4c$: similarly, $5b - 3b = 2b$: and $7a - 3a = 4a$.

On verifying the result by substituting the given values, the upper line becomes 78 and the lower line 40: the difference is 38, and this is also seen to be the value by substitution in the remainder $4a + 2b + 4c$.

The result obtained in subtraction may be verified by complementary addition, as in Arithmetic. Thus, in the last example, on adding the second and third lines together the result is found to be the same as the upper line.

The same process is applicable when positive and negative signs occur in the given expressions.

EXAMPLE 3. *Subtract* $2x^2 - 3xy - 4y^2$ *from* $7x^2 - xy - 6y^2$. *Verify when* $x = 2$, $y = 3$.

$7x^2 - xy + 6y^2$	$7 \times 4 - 2 \times 3 + 6 \times 9 = 76$
$2x^2 - 3xy + 4y^2$	$2 \times 4 - 3 \times 2 \times 3 + 4 \times 9 = 26$
$5x^2 + 2xy + 2y^2$	50

Mentally change the signs of all the terms in the second row. Now add $6y^2$ to $-4y^2$ or $6y^2 - 4y^2 = 2y^2$: in the next term $+3xy - xy = 2xy$, and finally $7x^2 - 2x^2 = 5x^2$.

Substitute the given values in the remainder $5x^2 + 2xy + 2y^2$; the sum is found to be 50, and this is also the value obtained by substituting in the two given expressions. The result may be checked by noting that the sum of the second and third lines is equal to the first line.

Brackets.--What are known as **brackets** are largely used to combine two or more terms and in this manner to simplify the various operations of addition, multiplication and other processes.

EXAMPLE 1. $9+(7-3)=13$.

The expression on the left of the equality sign indicates that 9 is to be added to the difference between 7 and 3, and this operation may be expressed by $9+7-3=13$.

EXAMPLE 2. $9-(7-3)=5$.

Here 9 is to diminished by the difference between 7 and 3. The result will be the same whether we subtract 4 from 9 or subtract 7 from 9+3. Hence, we may remove the brackets and write $9-(7-3)$ as $9-7+3$.

That is to say, when a minus sign occurs in front of a bracket, the signs of all the terms inside the bracket are to be changed when the brackets are removed. In the case of a plus sign being outside the bracket the signs of all the terms inside the bracket remain unaltered.

EXAMPLE 3. $9-(7+3)$.

This denotes that the sum of 7 and 3 is to be subtracted from 9. On removing the brackets $9-(7+3)$ becomes $9-7-3=-1$.

EXAMPLE 4. $a-(b-c)$.

This denotes that a is to be diminished by the difference between b and c; or b is to be subtracted from the sum of a and c.

When the brackets are removed the result becomes $a-b+c$.

If $a=4,\ b=3,\ c=2$.

Then $a-(b-c)=4-(3-2)=3$,

or $a-b+c=4-3+2=3$.

Similarly, $a+(b-c)$ denotes that the difference between b and c is to be added to a; this is obviously the same as subtracting c from the sum of a and b. When the brackets are removed the expression becomes $a+b-c$. These results may be verified by substitution as in the preceding examples.

MISCELLANEOUS EXERCISES V.

1. How much has a man left out of £500 after spending £57. 13*s*. 4½*d*.?

2. What must be added to £167. 15*s*. 7½*d*. to make £375. 14*s*. 6*d*.?

3. What will be the cost of 28 acres of land if the price per acre is £56. 3*s*. 6*d*. ?

4. If £126. 7*s*. be divided equally amongst 28 persons, how much will each receive ?

5. What sum of money will be required to give £5. 2*s*. 6*d*. to each of 35 persons ?

6. If 30 sheep cost £118. 3*s*. 6*d*., what is the cost per sheep ?

7. If the cost of a cablegram is 9*d*. per word, how many words can be sent for £1. 7*s*. 7*d*. ?

8. Find the cost of an army of 50 soldiers for one year, if the cost of each man is £73. 7*s*. 6*d*. per year.

9. The cost of a carpet and piano is £63. 5*s*. 6*d*. If the cost of the piano is £54. 14*s*. 9*d*., what is the cost of the carpet ?

10. If a man's stride is 2 ft. 8 in. long, how many steps will he take in a mile of 5280 ft. ?

11. If each ton of ore yields 890 grains of gold, how much gold will be obtained from 293 tons of ore ? [1 lb. = 7000 grains.]

12. A man earns £70 in a year of 300 days. What does this amount to per day ?

13. If one cwt. of iron is made into a length of a quarter of a mile of wire and costs £1. 1*s*., what will be the cost of 20 yards of wire and what will be the weight of 1000 yards ?

14. If a man takes 2306 steps in a distance of a mile, what is the average length of a stride ?

15. There are x oranges in one basket and y in another : how many are there altogether ? Verify when $x=30$, $y=50$.

16. A boy who is now x years of age was born when his father was n years old. What is the age of the father ? Verify when $x=15$, $n=35$.

17. A mother is b years older than her boy, who is now 4 years old. If the father is x years older than the mother, find the age of the father. Verify when $b=20$ and $x=7$.

18. A man earns three times and a woman twice as much as a boy. If a boy is paid at the rate of 3*d*. per hour, what will be the wages of 5 men, 4 women and 2 boys at 8 hours per day for 6 days ?

19. The fees for a term in a school amount to £880. If 130 of the students pay £4 per term and the remainder £3 per term, find the total number of students.

20. Write in a condensed form

(i) $(a\times a)\times a\times a\times a$; (ii) $a\times a\times a\times b\times b$.

21. Write out at full length (i) a^2b^2, (ii) a^2b^5, (iii) $a^2b^3x^4y^2$. Find the value in each case when $a=1$, $b=2$, $x=3$, $y=4$.

22. Express algebraically : the product of x and y divided by m is equal to n. Find the value of n when $x=6$, $y=3$, $m=9$.

23. A boy is n years old and his father is m years more than four times as old. What is the age of the father? Verify when $n=10$, $m=4$.

24. Subtract 5 from x.

25. From 5 take x.

26. The sum of two numbers is 20. If one is x, what is the other?

27. What is the product of b and 9?

28. The difference of two numbers is 9. If one is x, what is the other?

Remove brackets and simplify the following expressions:

29. $9+(5-3)$.

30. $9-(5-3)$.

31. $9-(7-3)$.

32. $5x-(2x+p)$.

33. $5x^2-(2x+p)$.

34. $5x^2-(2x-p)$.

35. $4x^2-(2x^2+3x)$.

MEASURES OF LENGTH.

Measurement of Length.—Draw two lines AD and EF; from A along AD mark off portions AB, BC, and CD each equal to EF; then AD is three times the length of EF.

If EF is one yard in length, then AD will denote 3 yards; and if EF is one inch in length, then AD will be 3 inches. The length EF is called a **unit** or a **standard.**

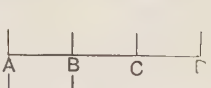


FIG. 2.

It is of the utmost importance that an invariable unit shall be used in measuring lengths; hence, in this and in other countries, certain suitable lengths called standards or units of length have been adopted.

The most important units of length are the **British** unit called the **yard**, and the **Metric** unit called the **metre.**

Smaller units are obtained by dividing the yard into three equal parts, each of which is called a **foot.** The foot is again subdivided into twelve equal parts, each part being known as an **inch.**

Inches are further subdivided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., as well as in many cases into tenths.

Dimensions are frequently indicated by marks. Thus, 3 feet 6 inches may be written as 3′ 6″.

British Measures of Length.

1 yard = 3 feet = 36 inches.
1 foot = 12 inches.
2 yards = 1 fathom.
5·5 yards = 1 rod or pole.
4 rods or 22 yards = 1 chain.
40 poles or 220 yards = 1 furlong.
80 chains = 1 mile.
8 furlongs, 1760 yards, 5280 feet = 1 mile.

Surveyors use the **chain** of 22 yards: each chain being equal to 100 **links.**

The following abbreviations are usually employed:

yd. denotes a yard or yards,
ft. „ a foot or feet,
in. „ an inch or inches,
ro. „ a rod or rods,
po. „ a pole or poles,
ch. „ a chain or chains.

Decimals.—To measure the lengths of the three lines *AB*, *CD* and *EF* (Fig. 3), we may use a scale in which the unit (1 inch) is divided into ten equal parts, when the distance from *A* to the first division is $\frac{1}{10}$th inch.

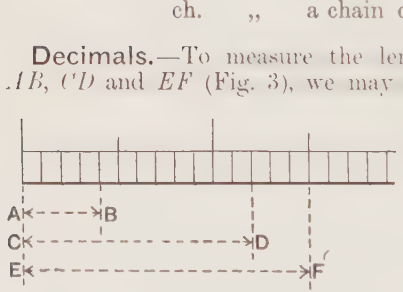


Fig. 3.

The length of *AB* is seen to be four of these divisions and is $\frac{4}{10}$, the length of *CD* is $1\frac{2}{10}$ and of *EF* is $1\frac{5}{10}$. Instead of the number $1\frac{5}{10}$ a dot (·) called the **decimal point**, or simply the **point**, is placed immediately after the units figure. Thus, the number $1\frac{5}{10}$ is written as 1·5. The point denotes that the 1 represents one unit, the 5 denotes tenths, and the number 1·5 is read as one decimal five or as *one point five.* In this manner the length of *CD*

is written 1·2 and the length of AB as ·4 or 0·4. These numbers are known as decimal fractions.

In the number 31·5 the point indicates that 1 represents units, the 3 tens, and the 5, tenths of the unit. Hence, just as the digits to the left of the decimal point represent units, tens, hundreds, thousands, etc., ascending by tens as we proceed step by step to the left, so the digits to the right of the decimal point denote tenths, hundredths, etc., descending by tenths as we proceed to the right.

A number such as 3·3 indicates a whole number of 3 units plus a decimal fraction of 3 tenths, or $3 + \frac{3}{10}$.

Similarly 3·33 denotes $3 + \frac{3}{10} + \frac{3}{100}$.

In such a series of figures as 33333·3333 each figure – reading from left to right—has one-tenth the value of the preceding figure. These facts may be expressed in tabular form.

Ten thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Tens of thousandths.
3	3	3	3	3	• 3	3	3	3

Addition, Subtraction, and Multiplication of Decimals.—The addition and subtraction of decimal fractions are effected in the same way as in the case of whole numbers and will therefore present no difficulty. The only necessary precaution is to arrange the numbers so that the unit figures, or the decimal points, are in the same vertical line.

Example 1. *Add together* 21·7, 13·21, 15·721, 3·815, 9·813, 0·814, 0·316, 0·091.

21·700
13·210
15·721
3·815
9·813
0·814
0·316
0·091

65·48

EXAMPLE 2. *Subtract* 10·943 *from* 54·446.

$$\begin{array}{r} 54{\cdot}446 \\ 10{\cdot}943 \\ \hline 43{\cdot}503 \end{array}$$

EXAMPLE 3. *Add together* 9·743, 8·526 *and* 0·405 *and from the sum subtract* 7·305.

$$\begin{array}{r} 9{\cdot}743 \\ 8{\cdot}526 \\ 0{\cdot}405 \\ \hline 18{\cdot}674 \\ 7{\cdot}305 \\ \hline 11{\cdot}369 \end{array}$$

EXAMPLE 4. *Multiply* 7·4 *by* 4·6.

$$\begin{array}{r} 74 \\ 46 \\ \hline 444 \\ 296 \\ \hline 3404 \end{array}$$

The process resembles *ordinary simple multiplication*. To fix the position of the decimal point, it is seen that 7·4 lies between the whole numbers 7 and 8, and 4·6 lies between the numbers 4 and 5. Hence the product $7{\cdot}4 \times 4{\cdot}6$ lies between 7×4 and 8×5 or between 28 and 40. Hence the units figure is the second figure in the product going from left to right, and the product is 34·04.

This method of determining the position of the decimal point is not always convenient, and the following rule is used: *To find the number of decimal places in the product, add the number of decimal places in the multiplier to those in the multiplicand.*

EXAMPLE 5. *Multiply* 13·45 *by* 4·37.

The product is 587765. As in Ex. 4, the product lies between 52 and 70, and is 58·7765, or, using the rule, two decimal places in the multipler and two in the multiplicand give four decimal places in the product, and the result is 58·7765.

EXERCISES VI.

Add together:

1. 0·078, 824·704, 65065·05, 9·034, 9290, 706·888, 10673·009, 7·777, 25·57.
2. 4·30726, 0·076428, 371·864, and 20·0472.
3. 24·5604, 0·07365, 407·8604, and 1·00753.
4. 3·08591, 0·265832, 154·098, and 17·9037.
5. 819·065, 973·72143, 0·006153, and 29·3482.
6. 3·9064, 0·08456, 327·265, and 14·00983.
7. 0·90807, 6·0543, 21·99, and 0·00068.

Add together:

8. 13·0009, 4·5672, 1·89, and 0·007999.
9. 25·00325, 1024·72. 0·785 and 20·02.

Subtract:

10. 875·589 from 20083·67.
11. 5·3804 from 100.
12. 33·1776 from 100.
13. 47·063782 from 710·04681.
14. 39·983417 from 840·8946.
15. 34·2097834 from 235·1937.
16. 53·94306 from 60·1531.
17. 4·60439 from 51·07.
18. The rise of a barometer on four consecutive days is found to be 0·135, 0·044, 0·092 and 0·573 inch respectively, and its fall during the following three days is 0·021, 0·417 and 0·012 in. If its height at first were 29·24 ins., what will be the last reading?

Metric Measurement.—The metric system, with its simple subdivisions into tenths, is a direct application of the decimal notation.

The unit of length, the **metre**, is the length of a certain bar of metal preserved at Sèvres near Paris. Copies of the standard metre are to be found in our own and other countries.

The metre is divided into ten equal parts each called a **decimetre**; a decimetre is divided into ten equal parts each a **centimetre** and a centimetre into ten equal parts each a **millimetre**.

Thus, a decimetre is one-tenth, a centimetre one hundredth and a millimetre the one-thousandth of a metre.

A length of 2 metres, 4 decimetres, 6 centimetres and 3 millimetres may, in accordance with the decimal notation, be written as 2·463 metres, or 246·3 centimetres or 2463 millimetres.

The relations between the metre and its subdivisions are as follows:

1 metre = 10 decimetres = 100 centimetres
= 1000 millimetres.
1 decimetre = 10 centimetres = 100 millimetres.
1 centimetre = 10 millimetres.

The *multiples* of the metre are:

10 metres = 1 **decametre.**
100 ,, = 10 decametres = 1 **hectometre.**
1000 ,, = 100 decametres = 10 hectometres
= 1 **kilometre.**

The following abbreviations for these units are usually employed:

kilom.	denotes	kilometres.
m.	,,	metres.
dcm.	,,	decimetres.
cm.	,,	centimetres.
mm.	,,	millimetres.

The conversion of a given number of kilom., etc., into metres, decimetres and so on, is effected by changing the position of the decimal point.

EXAMPLE 1. *Express* 0·00246 *kilom. as metres, etc.*

0·00246 kilom. = 2·46 m. = 24·6 dcm.
= 246 cm. = 2460 mm.

One end of a steel scale is shown in Fig. 4, having its upper edge divided into centimetres and millimetres and the lower

FIG. 4.—Steel scale.

into inches; it will be seen that 10 cm. are nearly equal to 4 inches.

It is frequently necessary to convert lengths expressed in one system into the corresponding values in the other. The following simple approximate relations should be learnt by the student:

1 foot = 30 centimetres.
1 inch = $2\frac{1}{2}$ centimetres.
35 yards = 32 metres.
10 metres = 11 yards.
20 metres = 22 yards = 1 chain (the length of a cricket pitch).
5 miles = 8 kilometres.

For more accurate calculations the following relations between the two systems of measurement should be used:

Conversion of British to Metric Measures of Length.

1 inch = 2·54 centimetres.
1 foot = 30·48 centimetres = 0·3048 metres.
1 yard = 0·914 metre.
1 mile = 1609·3 metres = 1·6093 kilom.

Metric to British.

1 millimetre = 0·0394 inch.
1 centimetre = 0·394 inch.
1 metre = { 39·37 inches. / 3·281 feet. / 1·094 yards. }
1 kilometre = 0·621 mile.

Example 1. *How many millimetres are there in* 12 *inches?*
In 1 in. there are 25·4 mm.
Hence in 12 in. there are $25{\cdot}4 \times 12 = 304{\cdot}8$ mm.

Example 2. *Express* 18 *in. in millimetres, centimetres and metres.*
$18 \times 25{\cdot}4 = 457{\cdot}2$ mm. $= 45{\cdot}72$ cm. $= 0{\cdot}4572$ m.

EXERCISES VII.

1. Measure the length and width of this page (i) in inches and fractions of an inch, (ii) in centimetres and millimetres. Hence find the number of centimetres in an inch.

2. Which is the longer, a line, 5 in. or 13 cm. long? Express the difference in centimetres.

3. Draw a line 8 cm. long; find the difference between this line and a line 3 in. long.

4. Draw a line AB 4 in. long. Produce the line to C, making $BC = 12$ cm. Write down the length of the line AC in centimetres.

5. Complete the following table:

Diameter in inches,	15	22	27	47
(i) Diameter in millimetres,				
Velocity in feet per second,	130·9	81·7	61·7	108·6
(ii) Velocity in metres per second				

Squared Paper.—A sheet of paper ruled with horizontal and vertical lines, forming a series of small squares, is called **squared** or section paper. In some kinds of squared paper the distance apart of the lines is $\frac{1}{10}$th of an inch, and each fifth or tenth division is indicated either by a rather thicker line than the others or by a line of different colour, generally red or blue, so as to simplify the process of counting the divisions.

In other kinds of squared paper the lines are at a distance of a millimetre apart and each centimetre is indicated by a thicker or darker line.

Many problems difficult, if not impossible, to solve by other methods, may be solved by the use of squared paper.

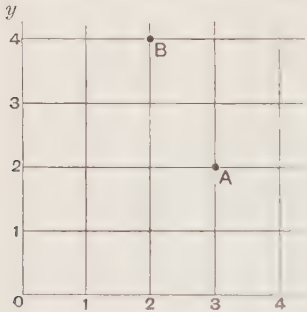


FIG. 5.—Plotting on squared paper.

Two lines at right angles, as Ox and Oy (Fig. 5), on the squared paper, are taken as axes of reference; the point of intersection O is called the **origin**.

To locate the position of any point, such as B (Fig. 5), we may draw a line parallel to the axis of y, meeting the axis of x in the point 2, and a line parallel to the axis of x, meeting the axis of y at 4. We can reach the point B by moving along the axis of x to 2 and then along $2B$ parallel to the axis of y to B.

The point B is fixed when the values of x and y are given; x and y are called the **coordinates of the point** B, and B is called the point (x, y).

Starting at the origin O, the consecutive squares along Ox may be numbered 0, 1, 2, 3, ..., etc., as in Fig. 5, or any other convenient unit may be used. In a similar manner the consecutive squares in the vertical direction (or along the axis of y) may be marked. Thus, if the coordinates of a point B are $x = 2$, $y = 4$, then B, the point of intersection of lines passing through 2 on Ox and 4 on Oy, is the point required. Similarly, A is a point $x = 3$, $y = 2$.

This process of marking the position of a point is known

as **plotting the point.** Its position may be marked by a small dot with a small circle round it, or a small cross may be used.

If the axes OX and OY are produced to OX' and OY' (as in Fig. 6) then four spaces, or quadrants, XOY, YOX', $X'OY'$ and $Y'OX$ are obtained.

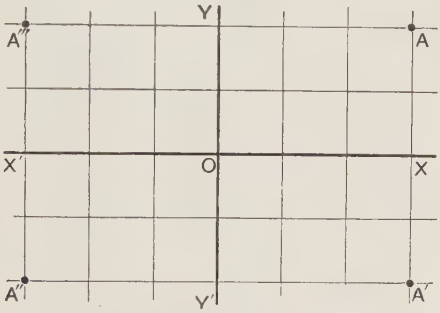


Fig. 6.

Values of x measured along OX or OX' are called **abscissae**, and are **positive** when drawn to the right in the direction OX, and **negative** when drawn to the left in the direction OX'.

Values of y measured along OY or OY' are called **ordinates** and are **positive** when above the axis XOX' and **negative** when below.

Thus, to plot the points (3, 2), (3, −2), (−3, −2) and (−3, 2), we proceed as follows: the first point A is in the positive quadrant, the point A' is positive for x and negative for y. A'' the point (−3, −2) and A''' the point (−3, 2) are obtained similarly.

Graphs.—When a series of numerical values of x and y are plotted, a corresponding series of points are obtained, and the line, straight or curved, drawn through the points is called a **graph.** It is easy to record statistics of various kinds by means of a curve. Also, it is possible to find from such a curve intermediate values not plotted. Such values are said to be obtained by **interpolation.**

EXAMPLE 1. *Show graphically the number of inches in a given number of centimetres. Find* (i) *the number of centimetres in* 3·5 *in.;* (ii) *the number of inches in* 11·43 *cm.*

1 in. = 2·54 cm. (p. 19),
6 in. = 15·24 cm.

Take any horizontal line on squared paper near the bottom left-hand corner. Select any convenient scale, *e.g.* each of the larger divisions measured horizontally on the squared paper may be taken to denote 5 cm., and a larger division measured on a vertical line may represent 1 inch.

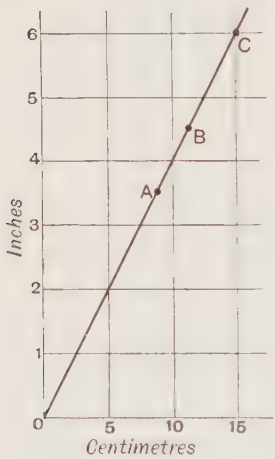


FIG. 7.—Conversion of inches to centimetres.

The point C, denoting 15·24 cm. horizontally and 6 in. vertically, is obtained; similarly the point B, representing 4·5 in. and 11·4 cm., is found. By joining C to B a line which passes through the origin O is obtained. Any number of inches up to 6 in. can now be read off from the sloping line OB.

(i) So that, corresponding to the vertical distance 3·5 at A, the corresponding number of centimetres is easily read off in a horizontal direction as 8·9; or 3·5 in. = 8·9 cm.

The values in (i) and (ii) are obtained by interpolation.

In many cases it is possible to obtain a value or values with some approach to accuracy in a plotted curve produced beyond the plotted points, such values are said to be obtained by **extrapolation.**

EXERCISES VIII.

1. Reduce 150 yds. to the decimal fraction of a kilometre.

2. How many lengths each 12·54 cm. can be cut from a strip of cloth 3·27 m. long, and what will be the length of the remaining piece?

3. Find the number of feet and metres in 0·5813 of a mile.

4. Express 14 in. in centimetres.

5. Reduce 3069·04 m. to the decimal fraction of a mile.

6. Find approximately the number of metres in the following:
(i) 1 inch, (ii) 1 ft., (iii) 1 yd., (iv) 1 mile.

7. Find (i) the number of metres in 107 ft., (ii) number of feet in 107 metres, (iii) number of kilometres in 105 miles.

8. Find the number of metres, decimetres and centimetres in 9 yds. 1 ft. 8·48 in.

9. What is the number of miles in 2414·016 metres?

10. Find the number of miles in 160·93 kilometres.

11. A yard of silk costs 5*s.* 4·3*d.*, what is the cost of the silk in francs per metre, given that a franc is equal to 9·38*d.*?

12. If a man makes 3068 steps, each step being a distance of 2 ft. 8 in., what distance will he walk in kilometres?

13. Obtain the number of metres in 6 miles 373 yds. 1 ft.

Addition and Subtraction of Vulgar Fractions.—Referring to Fig. 8, which represents one end of a foot-rule, it is seen that the unit distance AD, one inch, is divided into eight equal parts.

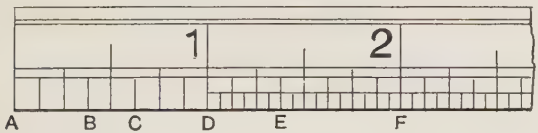


Fig. 8.—One end of a foot-rule.

The distance from A to B, which includes three of these parts, is called **three-eighths** of an inch, and is written $\frac{3}{8}$ or 3/8. The lower figure of the fraction $\frac{3}{8}$, called the **denominator**, indicates the number of equal parts into which the unit is divided; and the upper figure, 3, or the **numerator** of the fraction, denotes the number of parts supposed to be taken. A fraction of this kind, in which the denominator may be denoted by b and the numerator by a, is called the **vulgar fraction** $\frac{a}{b}$.

On reference to Fig 8, it will be seen that the unit distance DF is divided into 16 equal parts; and from D to E there are 6 equal divisions; hence the distance from D to E may be expressed as $\frac{6}{16}$ of the unit.

The fraction $\frac{6}{16}$ is obviously the same distance as the preceding distance from A to B and consequently $\frac{3}{8} = \frac{6}{16}$.

From simple illustrations of this kind it may be seen easily that $\frac{5}{8} = \frac{10}{16}$, $\frac{7}{8} = \frac{14}{16}$ and so on.

Or, the **numerical value of a fraction is unaltered when the denominator is multiplied by any convenient number, provided that the numerator is multiplied by the same number.**

Conversely, as the unit distance may be divided into any convenient number of equal parts, such as 4, 8, 16, 32, 64, ... when the denominator of a given fraction is divided by any convenient number, the value of the fraction will remain unaltered if the numerator is divided by the same number.

EXAMPLE 1. *Find the sum and difference of* $\frac{7}{16}$ *and* $\frac{3}{8}$.

By multiplying both numerator and denominator by 2, the second fraction is converted into the equivalent fraction $\frac{6}{16}$.

Hence $\frac{7}{16} + \frac{3}{8} = \frac{7}{16} + \frac{6}{16} = \frac{13}{16}$.

Also $\frac{7}{16} - \frac{3}{8} = \frac{7}{16} - \frac{6}{16} = \frac{1}{16}$.

The same methods are applicable when it is required to make a **comparison of fractions.**

EXAMPLE 2. *Which of the two fractions* $\frac{7}{64}$ *or* $\frac{1}{8}$ *is the greater?*

By multiplying the numerator and denominator of the second fraction by 8, it becomes $\frac{8}{64}$; hence it is clear that $\frac{1}{8}$ is greater than $\frac{7}{64}$ and that the difference is $\frac{1}{64}$.

EXAMPLE 3. *Add together* $\frac{7}{8}$ *and* $\frac{3}{64}$.

$$\frac{7}{8} = \frac{56}{64}.$$

Hence $\frac{7}{8} + \frac{3}{64} = \frac{56+3}{64} = \frac{59}{64}$.

EXAMPLE 4. *Add together* $\frac{7}{8}$ *and* $\frac{3}{16}$.

$$\frac{7}{8} = \frac{14}{16}.$$

$$\therefore \frac{7}{8} + \frac{3}{16} = \frac{14+3}{16} = \frac{17}{16}.$$

In the answer to Example 4, the numerator is greater than the denominator of the fraction, and the fraction is called an **improper fraction**. By dividing the numerator by the

denominator, the fraction may be written in the form of a **mixed number**, in this case $1\frac{1}{16}$.

EXAMPLE 5. *Find the sum of* $1\frac{1}{8}$, $2\frac{3}{16}$, *and* $3\frac{1}{4}$.

Here the sum of the whole numbers in the given fractions is

$$1+2+3=6.$$

The sum of the fractional parts is

$$\frac{1}{8}+\frac{3}{16}+\frac{1}{4}=\frac{2+3+4}{16}=\frac{9}{16}.$$

Hence $$1\tfrac{1}{8}+2\tfrac{3}{16}+3\tfrac{1}{4}=6\tfrac{9}{16}.$$

EXAMPLE 6. *Add together* $1\frac{1}{16}$, $\frac{3}{4}$, $\frac{7}{8}$ *and* $3\frac{1}{64}$.

The sum of the whole numbers is $3+1=4$.

,, ,, fractional parts is $\frac{1}{16}+\frac{3}{4}+\frac{7}{8}+\frac{1}{64}$

$$=\frac{4+48+56+1}{64}=\frac{109}{64}=1\tfrac{45}{64}.$$

Hence $$1\tfrac{1}{16}+\tfrac{3}{4}+\tfrac{7}{8}+3\tfrac{1}{64}=5\tfrac{45}{64}.$$

It is an instructive exercise to verify the preceding examples by means of the foot rule.

The work involved in simplifying a series of fractions is, in some cases, lessened by first simplifying two or more of the fractions and afterwards simplifying the remainder.

EXAMPLE 7. *Find the sum of* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ *and* $\frac{1}{7}$.

By simplifying four of the terms we obtain

$$\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{6}=\tfrac{6}{12}+\tfrac{4}{12}+\tfrac{3}{12}+\tfrac{2}{12}$$
$$=1\tfrac{5}{12}=\tfrac{5}{4}.$$

Combining this result with another of the fractions, we have

$$\frac{5}{4}+\frac{1}{5}=\frac{25+4}{20}=\frac{29}{20}.$$

Finally, $$\frac{29}{20}+\frac{1}{7}=\frac{203+20}{140}=\frac{223}{140}.$$

Hence $$\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5}+\tfrac{1}{6}+\tfrac{1}{7}=2\tfrac{23}{140}.$$

The product of the denominators of the given fractions could be used as a common denominator, but the numbers involved are usually greater than necessary; hence the **lowest number into which each denominator will divide without**

a remainder is obtained. This number is called the **Lowest Common Multiple** of the given numbers.

The Lowest Common Multiple, usually denoted by the letters L.C.M., of two or more given numbers may be obtained by the application of a simple rule, or in some cases by inspection.

EXAMPLE 8. *Simplify* $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.

In this case the L.C.M. is seen by inspection to be 12.

$$\therefore \frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6+4+3}{12}=\frac{13}{12}.$$

Or, we may arrange the respective denominators in a horizontal line and, dividing by 2, the numbers 3 and 2 are obtained. The L.C.M. is the product of the numbers in the lowest line and the divisor; *i.e.* the L.C.M. is

$$2 \mid 2 . 3 . 4$$
$$3 . 2$$

$$3 \times 2 \times 2 = 12.$$

Having found the L.C.M., the numerators of the respective fractions are obtained most easily by writing the factors of the common denominator instead of the product.

Thus, as the denominator of the first fraction is 2, the numerator is multiplied by the remaining factors 2 and 3, giving 6 as a product. In like manner, the remaining numerators are 4 and 3.

EXAMPLE 9. *Simplify* $\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{18}-\frac{1}{16}$.

Arrange the denominators in a horizontal line, as in Example 8.

The factors of the L.C.M are seen to be

$$3 \times 2 \times 2 \times 5 \times 3 \times 4,$$

and the given fractions may be written

$$3 \mid 3, \ 4, \ 5, \ 18, \ 16$$
$$2 \mid 4, \ 5, \ 6, \ 16$$
$$2 \mid 2, \ 5, \ 3, \ 8$$
$$5, \ 3, \ 4$$

$$\frac{240+180-144+40-45}{3 \times 2 \times 2 \times 5 \times 3 \times 4}=\frac{271}{720}.$$

EXAMPLE 10. *Add together* $\frac{1}{2}, \frac{1}{3x}, \frac{1}{4x}$.

The L.C.M. is here $12x$. Hence, the given fractions become

$$\frac{6x+4+3}{12x}=\frac{6x+7}{12x}.$$

EXAMPLE 11. *Subtract* $\frac{1}{3x}$ *from* $\frac{1}{2x}$.

The L.C.M. is $6x$; $\therefore \frac{1}{2x}-\frac{1}{3x}=\frac{3-2}{6x}=\frac{1}{6x}$.

EXAMPLE 12. *Add* $\frac{1}{x-3}$, $\frac{1}{x-4}$.

The common denominator is the product of the two denominators;

$$\therefore \frac{1}{x-3}+\frac{1}{x-4}=\frac{x-4+x-3}{(x-3)(x-4)}=\frac{2x-7}{x^2-7x+12}.$$

EXAMPLE 13. *Subtract* $\frac{3}{x+6}$ *from* $\frac{2}{x-5}$.

$$\frac{2}{x-5}-\frac{3}{x+6}=\frac{2(x+6)-3(x-5)}{(x-5)(x+6)}$$

$$=\frac{2x+12-3x+15}{(x-5)(x+6)}=\frac{-x+27}{x^2+x-30}.$$

EXAMPLE 14. *Reduce to its simplest form* $\frac{x-2y}{4}-\frac{x+2y}{5}+\frac{5y}{6}$.

Find the value when $x=5$, $y=3$.

The L.C.M. of the denominator is 60. Hence, the given fractions become

$$\frac{15x-30y}{60}-\frac{12x+24y}{60}+\frac{50y}{60}=\frac{3x-4y}{60}.$$

Substituting the given values;

$$\therefore \frac{15-12}{60}=\frac{1}{20}.$$

EXERCISES IX.

Add together

1. $\frac{1}{2}$ and $\frac{1}{3}$.
2. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
3. $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{6}$.
4. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{7}{24}$ and $1\frac{7}{40}$.
5. $1\frac{2}{3}$, $4\frac{5}{6}$, $3\frac{4}{5}$, $2\frac{3}{4}$ and $1\frac{3}{20}$.
6. $\frac{1}{18}$, $4\frac{1}{6}$, $\frac{1}{15}$, $\frac{1}{9}$ and $1\frac{3}{5}$.
7. $\frac{1}{21}$, $3\frac{7}{15}$, $\frac{3}{5}$, $\frac{5}{7}$ and $3\frac{3}{105}$.
8. $\frac{2}{5}$, $\frac{19}{35}$, $\frac{3}{4}$, $\frac{1}{7}$ and $1\frac{3}{140}$.
9. $2\frac{7}{8}$, $\frac{13}{14}$, $\frac{19}{24}$, $5\frac{3}{7}$ and $2\frac{2}{42}$.
10. $\frac{11}{12}$, $3\frac{17}{28}$, $5\frac{8}{9}$, $2\frac{31}{63}$ and $1\frac{2}{21}$.
11. $4\frac{37}{60}$, $3\frac{2}{9}$, $2\frac{13}{20}$, $1\frac{4}{5}$ and $3\frac{2}{45}$.
12. $\frac{a}{x}$, $\frac{a}{2x}$, $\frac{a}{3x}$.

Subtract

13. $2\frac{1}{4}$ from $20\frac{1}{12}$.
14. $3\frac{8}{11}$ from $7\frac{2}{5}$.
15. $18\frac{38}{91}$ from $24\frac{2}{13}$.
16. $8\frac{5}{13}$ from $9\frac{9}{12}$.
17. $15\frac{11}{24}$ from $19\frac{1}{8}$.
18. $17\frac{13}{34}$ from $18\frac{11}{51}$.
19. $2\frac{4}{7}$ from $5\frac{5}{54}$.
20. $\frac{1}{2a}$ from $\frac{1}{6ac}$.

21. The length of a rope is 50 yards. What length will remain when three pieces $17\frac{2}{7}$ yds., $3\frac{1}{9}$ yds. and $12\frac{1}{4}$ yds. respectively have been cut off?

22. Subtract $\frac{3}{11} \times 1\frac{3}{4}$ cwt. from $3\frac{1}{7}$ tons.

23. A man owes £4, and pays at various times the following amounts: 10*s.*, 15*s.*, £$\frac{4}{5}$, £$1\frac{9}{20}$. How much will he still owe?

24. Simplify $\dfrac{1}{a+b}+\dfrac{2b}{a^2-b^2}+\dfrac{1}{a-b}$.

Find the value when $a=4$, $b=1$.

25. Simplify $\dfrac{7a-4}{3}-\dfrac{11a-7}{5}$.

Find the value when $a=5$.

Simplify the following expressions:

26. $\dfrac{x-2y}{4}+\dfrac{x+2y}{5}+\dfrac{5y}{6}$.

27. $\dfrac{1}{3}(x-3y)+\dfrac{2x-y}{2}-\dfrac{1}{12}(7x-9y)$.

Find the value when $x=3$, $y=1$.

28. $\dfrac{45a^3x}{36a^2x^3y}$, when $a=4$, $x=3$, $y=1$.

29. $\dfrac{a^3-x^3}{a^2-x^2}$, when $a=4$, $x=3$.

30. $\dfrac{10x^2}{(1+x^2)(1-4x^2)}+\dfrac{2}{1+x^2}-\dfrac{1}{1-2x}$, when $x=2$.

31. $\dfrac{1}{x+2}+\dfrac{2}{x+3}$.

32. $\dfrac{1}{x-2}+\dfrac{2}{x-3}$.

33. $\dfrac{209ab^3x^2}{133a^2bx^4}$, when $a=4$, $x=3$, $b=2$.

34. Write algebraically: Multiply the cube of a by the square of b and divide the product by the square root of c. Find the value when $a=2{\cdot}5$, $b=3{\cdot}2$ and $c=4$.

35. Write in words:

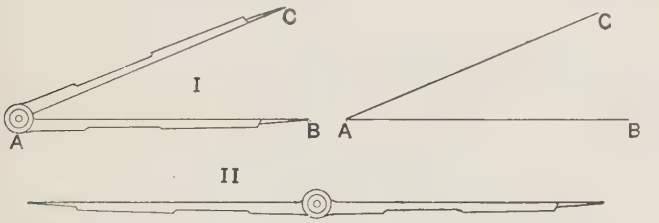
(i) $a^4b^2c^3$; (ii) $x^2y^3z^5$; (iii) $m^3n^2 \div \sqrt{p}$.

36. Write algebraically: The fourth power of x multiplied by the square root of y divided by the cube of z. Find the value when $x=2$, $y=9$, $z=2{\cdot}5$.

CHAPTER II.

Measurement of Angles. Construction of Plane Figures. Measurement of Area. Area of Rectangle. Vulgar Fractions: Multiplication, Division. Conversions and Simplifications. Metric Measures of Area. Decimals: Multiplication, Contracted Methods; Division, Contracted Methods. Decimals of Concrete Quantities. Averages.

Measurement of Angles.—If two lines BA and AC (Fig. 9) intersect at a point A, the inclination of one to the other is called the **angle BAC**, or simply the **angle A**.



Assuming one of the limbs of a pair of compasses, or dividers, as AB (Fig. 9), to be held in a horizontal position, then, on rotating the remaining limb about the centre A (and in the opposite direction to the hands of a watch), an angle BAC is obtained. If AC is perpendicular to AB, as at III., the angle BAC is a right angle. If AC

FIG. 9.—I. Acute angle, II. angle of 180°, and III. a right angle.

is in the line BA produced, as at II. (Fig. 9), the angle BAC is equal to two right angles.

If AC is a line which can be rotated about a centre A in the opposite direction to the minute hand of a clock, then when AC coincides with AE the angle is a right angle. If the arc BE be divided into 90 equal parts, the straight lines joining any two consecutive points to the centre A will enclose a **unit angle**, or an angle of **one degree**.

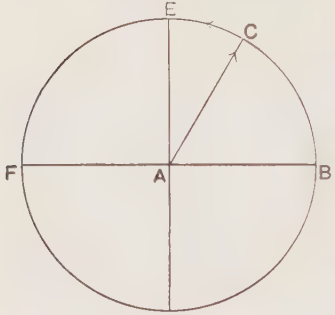


Fig. 10.

When AC coincides with AF the angle turned through measures 180 degrees, written as 180°.

When AC makes a complete turn, or rotation, the angle is four right angles or 360°.

The angle BAC would be indicated by stating the number of degrees in the angle.

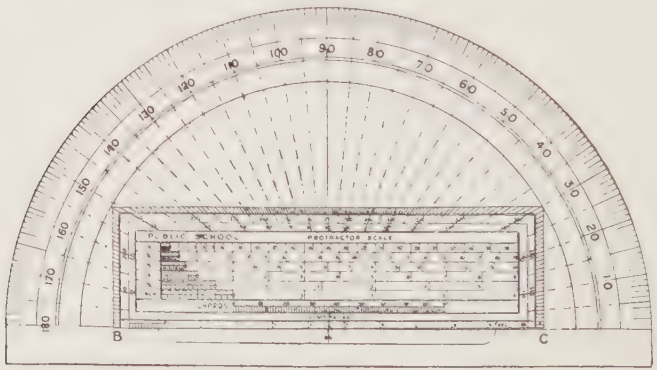


Fig. 11.

Plotting of Angles.—Angles may be set out by using some form of **protractor**.

Protractors are made in various forms; the **semicircular** and **rectangular** are in general use. The angles are in each case marked along the edge of the instrument.

Thus, if it is required at a point A to set out an angle of 35° with the line AB.

The protractor is placed with the mark $*$ at A; and its edge is made to coincide with AB, and a mark is made corresponding to 35°. This mark joined to A by a straight line gives the angle required.

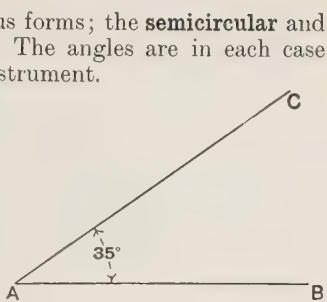


FIG. 12.

Conversely, if it is required to measure the magnitude of the angle between two given lines, the instrument is placed as before and the angle corresponding to the line AC, in the case of the angle BAC (Fig. 12), is noted.

Scale of Chords.—Another method which may be used to set out or to measure the magnitude of a given angle is to use a **scale of chords.**

EXAMPLE 1. *Set out an angle of* 25°.

Draw a horizontal line AB, and with A as centre and a radius of (say) 10 units draw an arc of a circle DF, intersecting AB in D.

In Table IV, there will be found a column of numbers marked *chords*; these give the numerical values of chords of angles from 0° to 90°. Opposite 25° the value of the chord of 25° is found to be 0·433. This is calculated to a radius unity, hence, as the radius of the arc is 10, the value of the chord is 4·33. With D as centre and with a radius of 4·33 describe an arc intersecting the arc DF in C. Join AC; then BAC is an angle of 25°.

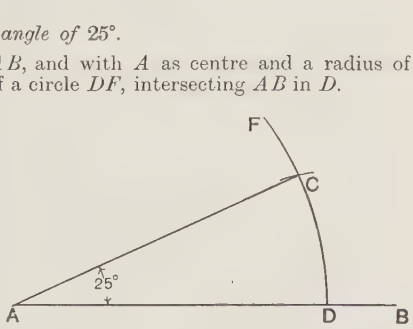


FIG. 13.

Conversely, given an angle, its magnitude can be ascertained by drawing an arc of 10 units, measuring to the same scale the length of the chord, and finally referring to Table IV.

PLANE FIGURES ENCLOSED BY STRAIGHT LINES.

Definitions.—The following right lined figures, or figures made up of straight lines, are of importance.

Rectangle. A four-sided figure, in which two pairs of opposite sides are parallel and the angles are right angles, is called a rectangle.

FIG. 14.

Square. When the sides of a rectangle are equal in length the figure is called a square.

FIG. 15.

Rhombus. If all the sides of a four-sided figure are equal, but the angles are not right angles, the figure is called a rhombus.

FIG. 16.

Parallelogram. When the opposite sides of a four sided figure are equal and parallel, but the angles are not right angles, the figure is called a parallelogram.

It should be noticed that the rectangle, square and rhombus, may be considered as special cases of parallelograms.

FIG. 17.

Trapezium. A four-sided figure in which only two of the sides are parallel is called a trapezium.

FIG. 18.

Quadrilateral. Any four-sided figure is called a quadrilateral.

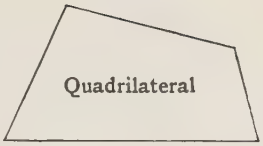


Fig. 19.

Triangle. A three-sided figure is called a triangle. Special names are given to various forms of triangles. Thus, when one of its angles is a right angle the figure is called a **right-angled triangle**. The side AB opposite the right angle is called the **hypotenuse**.

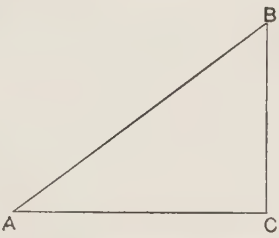


Fig. 20.—Right-angled triangle.

The relations between the sides of a right-angled triangle is that the square on the side AB opposite the right angle is equal to the sum of the squares on AC and CB containing the right angle.

The three angles of a triangle are usually denoted by the letters A, B, C, as shown in Fig. 20; the sides opposite the angles are denoted by corresponding small letters thus, AB is denoted by c, AC by b, and BC by a.

Hence, for a right-angled triangle, the relation between the sides is given by $c^2 = a^2 + b^2$.

Example 1. *Draw a triangle having its three sides* 3, 4 *and* 5 *units in length respectively. What is the angle between the two sides* 3 *and* 4?

Draw a triangle; make $AC = 4$ in.; with A as centre and radius 5 in., describe an arc of a circle, and with C as centre, radius 3 in., describe another arc intersecting the former in B. Join A and C to the point of intersection. Then the angle at C will be found to be a right angle.

Using the relation $c^2 = a^2 + b^2$ already stated we find $5^2 = 4^2 + 3^2$.

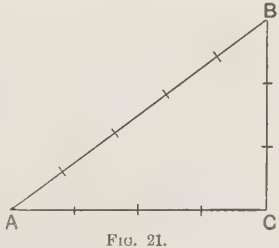


Fig. 21.

The units used may be of any magnitude, yard, foot, inch, centimetre, etc. This property, that in a right-angled triangle the three sides are as 3 is to 4 is to 5, is largely used by builders and others to construct what is known as a "square" Fig. 22.

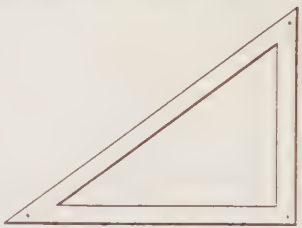


Fig. 22.

Equilateral triangle. A triangle is equilateral when each of its three sides are equal in length.

Isosceles triangle. When two of the sides of a triangle are equal in length, the figure is called an isosceles triangle.

The three angles of a triangle are together equal to two right angles. This fact may be verified by drawing a triangle

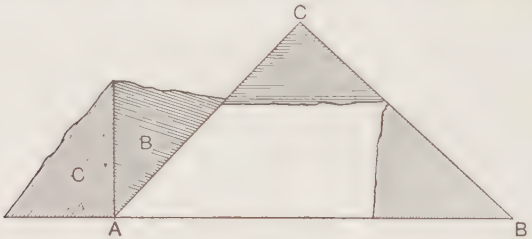


Fig. 23.—The three angles of a triangle are equal to 180°.

ABC and cutting out the three angles. If the angles B and C are added to A, one side of the angle C will coincide with the line BA produced.

An acute-angled triangle is a triangle in which each of the angles, as in Fig. 25, is less than 90°.

An obtuse-angled triangle is a triangle in which one angle is greater than 90°. Fig. 24.

Fig. 24.

EXAMPLE 2. *Draw a triangle having its three sides a, b, c, 3″, 2·75″ and 1·75″ respectively. Find the three angles.*

Draw the base $CB = 3$ in.; with C as centre and radius 2·75″, describe an arc; with B as centre and radius 1·75″, describe an arc intersecting the former in A. Join C and B to A. Then ABC is the triangle required. Measure the angles by means of the scale of chords, or by a protractor; then B is found to be 64°·6, C is 35°·1 and A is 80°·3.

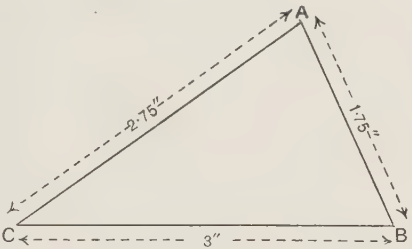


FIG. 25.—To construct a triangle having given the lengths of its three sides.

EXAMPLE 3. *Draw a triangle ABC, given $A = 60°$, $B = 54°$ and side $c = 100$ mm. Find the remaining angle and the two remaining sides.*

Set off $AB = 100$ mm.; make the angles A and B equal to 60° and 54° respectively. As the three angles of a triangle together equal 180°, the remaining angle $C = 180° - (60° + 54°) = 66°$. The side AC is found to be 88·55 mm. and BC is 94·8 mm.

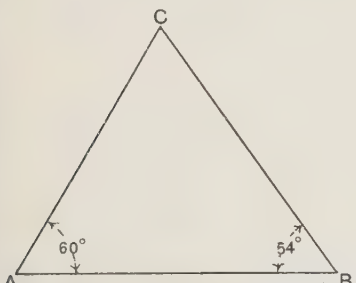


FIG. 26.—To construct a triangle given one side and two adjacent angles.

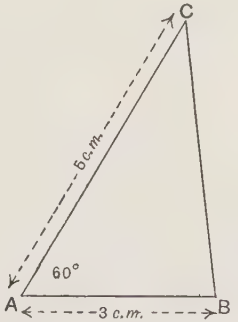


FIG. 27.—To construct a triangle given two sides and included angle.

EXAMPLE 4. *In a triangle ABC, $A = 60°$, $c = 3$ cm., $b = 5$ cm. Find the remaining angles and the remaining side.*

Make AB equal to 3 cm. and at A draw an angle of 60°. Make $AC = 5$ cm. and join C to B. Then the angle at B is found to be 83°·4, C is 36°·6 and the side BC is 4·36 cm.

Polygons.—All the figures now to be mentioned are known as polygons. In what is called a **regular polygon** all the sides are equal and the angles are equal.

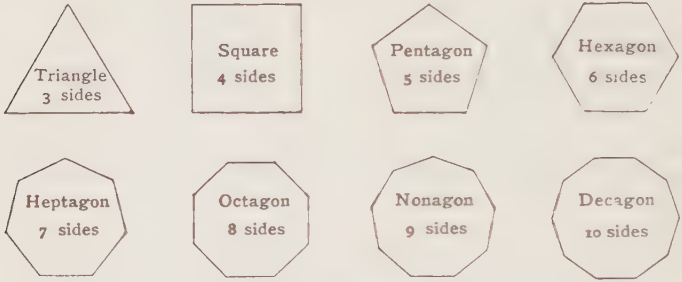


FIG. 28.

A regular polygon of three sides is an **equilateral triangle**: of four sides a **square**; of five sides a **pentagon**: of six sides a **hexagon**; of seven sides a **heptagon**: of eight sides an **octagon**; of nine sides a **nonagon**; and of ten sides a **decagon**.

EXERCISES X.

1. The sides of a triangle are 1·5, 3·6 and 3·9 ft. respectively. Find its angles.

2. The three sides of a triangle are 6·5, 7 and 7·5 ft. Find its angles.

3. Two sides of a triangle are 12·7 and 7·62 ft. respectively and the included angle is 50°. Find the two remaining angles and the length of the third side.

4. Two sides of a triangle are 13·1 cm. and 7·2 cm. and the included angle is 40°. Find the remaining angles and the remaining side.

5. The three sides of a triangle are 4·5, 5 and 5·5 inches respectively. Find its angles.

6. What are the three angles of a triangle in which the sides are 4 cm., 5 cm. and 6 cm. respectively.

AREA.

Measurement of Area.—The measurement of a given length is, as has been seen, obtained by finding the number of times it contains the unit of length. Similarly, in measuring areas a unit is required. The *unit of area is the area of a square whose side is the unit of length.*

The side of such a square may be a yard, a foot or an inch, according to the unit of length chosen; and the unit area would be in these cases 1 square yard, 1 square foot or 1 square inch respectively.

It should be noticed that the area of a figure has no reference to its **shape.** Thus, a figure may have an area of 1 square foot and yet be neither a square of 1 ft. side nor a rectangle.

British Measures of Area.

144 sq. in. = 1 sq. ft.
1296 sq. in. = 9 sq. ft. = 1 sq. yd.
4840 sq. yd. = 1 acre.
640 acres = 1 sq. mile.

Example 1. *Show graphically the number of square inches in a square foot.*

Mark off on squared paper from *E* to *F* (Fig. 29) 12 equal divisions and a similar number vertically from *E* to *H*. The two lines *EF* and *EH* with *FG* and *GH* form the square *EFGH*, enclosing 144 equal squares.

The side of each of these squares represents 1 inch, and the area of each square is 1 square inch. Hence the area of 1 sq. ft. = 144 sq. in.

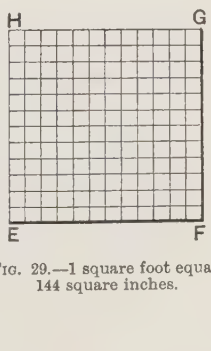


Fig. 29.—1 square foot equals 144 square inches.

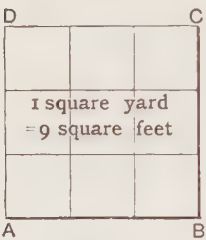


FIG. 30.

EXAMPLE 2. *Show graphically the number of square feet in a square yard.*

Mark off from A to B three equal divisions on squared paper and a similar number vertically. Let each division denote 1 foot.

The lines AB, BC, CD, DA will form a square $ABCD$; it will be seen that the number of squares enclosed within $ABCD$ is 9. Each of these smaller squares has sides of 1 ft. long; hence the area of a square yard is 9 sq. ft.

The measurement of land is usually estimated by using a **chain**, 22 yards long, divided into 100 equal parts called **links**. Areas are estimated by the **square chain**.

1 chain	= 22 yds. = 100 links.
1 sq. chain	= 484 sq. yds. = 10,000 sq. links.
10 sq. chains	= 4840 sq. yds. = 1 acre.
1 pole, perch or rod	= $5\frac{1}{2}$ yds.
1 sq. pole, sq. perch or sq. rod	= $30\frac{1}{4}$ sq. yds.
40 sq. poles	= 1 rood.
4 roods	= 1 acre.

EXAMPLE 3. *Show by means of squared paper the number of square yards in a square pole and a square chain.*

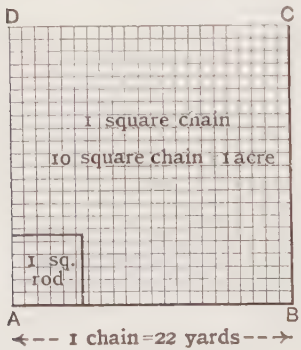


FIG. 31.

As in the preceding examples, make a square of 22 units side. Then the number of unit squares enclosed is seen to be $22 \times 22 = 484$ sq. yds.

The area of a square pole is obtained by making a square having each side $5\frac{1}{2}$ yds. The area is $5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$ sq. yds.

Area of a Rectangle.—Draw a line AB 4 inches long, and divide it into four equal parts. At the extremities A and B erect perpendiculars BC and AD, making BC and AD each one inch in length. Join DC; then $ABCD$ is a rectangle

Fig. 32.

of length 4 and breadth 1. If perpendiculars are drawn at the points of division 1, 2, 3, the rectangle $ABCD$ is divided into four equal squares, each one inch side, and the area of the figure $ABCD$ is the product of AB and AD, or 4 sq. in.

If the sides BC and AD are produced to F and G, making $CF = DG = 3$ inches, on joining F to G and drawing lines

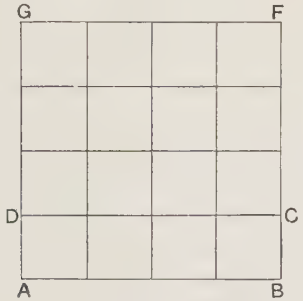


Fig. 33.—Area of a rectangle.

parallel to AB and BF, the figure $DCFG$ is seen to be divided into twelve equal squares, each one inch side. Hence the area of a rectangle with sides 3 in. and 4 in. equals $3 \times 4 = 12$ sq. in.

The **area of a rectangle** is obtained by **multiplying the number of units of length in its length by the number of units of length in its breadth.**

EXAMPLE 1. *The sides of a rectangle are* $2\frac{1}{4}$ *in. and* $2\frac{1}{2}$ *in. respectively. Find the area.*

Set off from A to D (Fig. 34) nine equal distances one-quarter of an inch apart; and from A to B five equal distances half an inch apart.

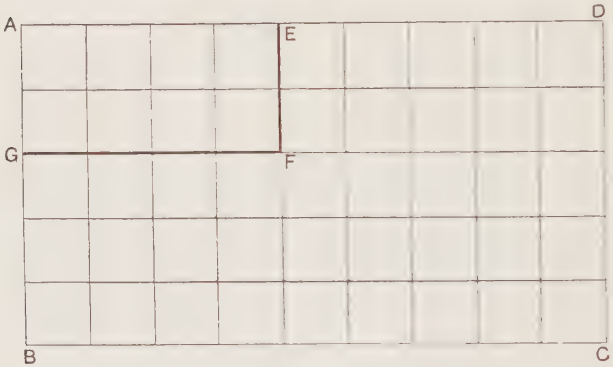


FIG. 34.

Complete the rectangle and draw lines through the points of division parallel to the sides.

The area of the rectangle is 9×5 but the area of 1 sq. in. ($AEFG$) is 4×2. Hence area of rectangle $= \frac{9 \times 5}{4 \times 2} = \frac{45}{8}$ sq. in.

As the only dimensions occurring in a rectangle are length and width, the dimensions are frequently written down with the sign of multiplication between them, or, sometimes the word "by" is used to indicate the size of the rectangle.

Thus, the data that the length of a rectangular opening is 6·75 ft. and its width 3·25 ft. may be abbreviated into "an opening is 6·75 ft. $\times$ 3·25 ft." or "an opening $6\frac{3}{4}$ ft. by $3\frac{1}{4}$ ft."

The dimensions of length and width may consist of whole numbers or may involve vulgar or decimal fractions. The area in each case is obtained by finding the product of the numbers (expressed in terms of the same unit, inches, feet or yards).

EXAMPLE 2. *The area of a rectangle is* 20 *sq. ft. and the length is* 5 *ft. What is the width?*

$$\text{Width} = \frac{20}{5} = 4 \text{ ft.}$$

EXAMPLE 3. *The area of a floor is* 12 *sq. yds. If its width is* 9 *ft., find its length.*

As 9 ft. = 3 yds.

$$\text{Length} = \frac{12}{3} = 4 \text{ yds.}$$

Multiplication of Fractions.—The process of multiplication of one fractional expression by another may be carried out in simple cases by means of a foot-rule.

EXAMPLE 1. *Multiply* $\frac{3}{16}$ *by* 4.

Refer to the rule (Fig. 8), and count four consecutive lengths of $\frac{3}{16}$; $\frac{12}{16}$ are obtained; but $\frac{12}{16} = \frac{3}{4}$.

Or, if we divide the denominator 16 by 4 the result, $\frac{3}{4}$, is obtained;

$\therefore \frac{3}{16} \times 4 = \frac{12}{16}$, the result $\frac{3}{4}$ is obtained.

From simple exercises of this kind we may obtain the following rule: **To multiply a fraction by a whole number, either multiply the numerator of the fraction by the number or divide the denominator by it.**

The last example could be written in the form $\frac{3}{16} \times \frac{4}{1}$. Hence, the product of two fractions may be obtained by **multiplying the numerators together to obtain the new numerator and the denominators for a new denominator.**

The word **of** is frequently used instead of the sign of multiplication and has the same meaning as "multiplied by."

Thus $\frac{3}{16}$ of $\frac{3}{4} = \frac{3}{16} \times \frac{3}{4} = \frac{9}{64}$.

EXAMPLE 2. $\frac{7}{16} \times \frac{3}{4} = \frac{21}{64}$.

EXAMPLE 3. $\frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$.

EXAMPLE 4. $\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$.

EXAMPLE 5. $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$.

EXAMPLE 6. $\frac{7}{8}$ of $\frac{3}{4} = \frac{21}{32}$.

EXAMPLE 7. $\frac{3}{8}$ of $\frac{3}{8} =$

EXAMPLE 8. $\frac{1}{2}$ of $\frac{3}{2} = \frac{3}{4}$.

Each of the preceding examples should be verified by means of a foot-rule.

EXAMPLE 9. $3\frac{1}{4} \times 5$.

As $3\frac{1}{4} = 3 + \frac{1}{4}$, then $3\frac{1}{4} \times 5 = (5 \times 3) + (5 \times \frac{1}{4}) = 16\frac{1}{4}$.

Or, convert the given mixed number into an improper fraction, thus,

$$3\frac{1}{4} = \frac{13}{4};$$

$$\frac{13}{4} \times 5 = \frac{65}{4} = 16\frac{1}{4}.$$

EXAMPLE 10. $3\frac{1}{4} \times 2\frac{1}{8}$.

$$3\frac{1}{4} \times 2\frac{1}{8} = \frac{13}{4} \times \frac{17}{8} = \frac{221}{32}$$
$$= 6\frac{29}{32}.$$

EXAMPLE 11. *Find the area of the top of a table if the length is 8 ft. 3 in. and the breadth 4 ft. 6 in.*

$$8 \text{ ft. } 3 \text{ in.} = 8\tfrac{1}{4} \text{ ft. and } 4 \text{ ft. } 6 \text{ in.} = 4\tfrac{1}{2} \text{ ft};$$
$$\therefore \text{ area} = 8\tfrac{1}{4} \times 4\tfrac{1}{2} = \frac{33}{4} \times \frac{9}{2}$$
$$= \frac{297}{8} = 37\tfrac{1}{8} \text{ sq. ft.}$$

EXAMPLE 12. *Find the cost of paving a court 30 ft. long and 2 ft. 6 in. wide at 2s. 3d. per square yard.*

$$\text{Area} = 30 \times 2\tfrac{1}{2} = 75 \text{ sq. ft.} = \tfrac{75}{9} \text{ sq. yds.}$$
$$\text{Cost} = \tfrac{75}{9} \times 2\tfrac{1}{4}s. = \tfrac{75}{4} = 18s.\ 9d.$$

Similar processes are applicable when fractions containing letters in addition to numbers are to be multiplied together.

EXAMPLE 13. $\frac{7x}{16} \times \frac{3y}{4}$.

Here the product of 7 and 3 is 21, and of 16 and 4 is 64; the product of x and y is $x \times y$ or xy. Hence

$$\frac{7x}{16} \times \frac{3y}{4} = \frac{21xy}{64}.$$

This may be verified; thus, when $x = 2$ and $y = 3$,

$$\frac{7x}{16} = \frac{14}{16} \text{ and } \frac{3y}{4} = \frac{9}{4};$$
$$\therefore \frac{7x}{16} \times \frac{3y}{4} = \frac{14}{16} \times \frac{9}{4} = \frac{63}{32},$$
$$\text{also } \frac{21xy}{64} = \frac{21 \times 2 \times 3}{64} = \frac{63}{32}.$$

EXAMPLE 14. $\frac{5x}{8y} \times \frac{3y}{8x} = \frac{15xy}{64xy} = \frac{15}{64}$.

EXAMPLE 15. $\frac{7b}{8a} \times \frac{1}{b} = \frac{7b}{8ab} = \frac{7}{8a}$.

Division of Fractions.—The simpler cases of the division of fractions may be verified by the foot-rule (Fig. 35).

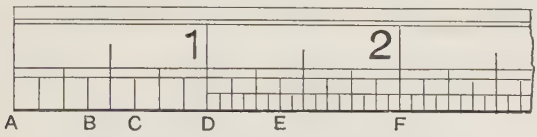


Fig. 35.

Example 1. *Divide* $\frac{3}{8}$ *by* 2.

This may be written as $\frac{3}{8} \div 2$.

The distance from A to B (Fig. 35) denotes $\frac{3}{8}$; dividing this by 2 we obtain a distance of $\frac{3}{16}$ in.

Thus, dividing by 2 is the same as taking one-half of the fraction.

Similarly, $\frac{3}{8}$ divided by 3 is the result of taking one-third of the fraction and would give a distance of $\frac{1}{8}$ in. In like manner $\frac{5}{8}$ divided 5 gives $\frac{1}{8}$.

Thus, dividing by 2 is equivalent to multiplying by $\frac{1}{2}$.

" " 3 " " " $\frac{1}{3}$.

" " 5 " " " $\frac{1}{5}$,

and the preceding fractions may be written

$$\frac{3}{8} \div 2 = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16},$$
$$\frac{3}{8} \div 3 = \frac{3}{8} \times \frac{1}{3} = \frac{1}{8},$$
$$\frac{5}{8} \div 5 = \frac{5}{8} \times \frac{1}{5} = \frac{1}{8}.$$

These examples may be verified as already indicated, and will serve to illustrate the following rule: **To divide one fraction by another invert the divisor and proceed as in multiplication.**

Example 2. *Divide* $\frac{5}{8}$ *by* $\frac{3}{8}$.

If we divide $\frac{5}{8}$ by 3 we obtain $\frac{5}{8} \times \frac{1}{3}$; but in obtaining this result the divisor used is 8 times as great as the given divisor, since 3 is eight times $\frac{3}{8}$. Hence the result obtained must be multiplied by 8;

$$\therefore \frac{5}{8} \div \frac{3}{8} = \frac{5}{8} \times \frac{8}{3} = \frac{5}{3}.$$

EXAMPLE 3. *Divide* $\frac{3}{64}$ *by* $\frac{3}{16}$.

$$\frac{3}{64} \div \frac{3}{16} = \frac{3}{64} \times \frac{16}{3} = \frac{1}{4}.$$

EXAMPLE 4. *The area of one side of a wall is* $51\frac{3}{4}$ *sq. ft. If the length is* 7 *ft.* 8 *in., what is the height?*

As 7 ft. 8 in. $= 7\frac{2}{3}$ ft.;

$\therefore$ height $= 51\frac{3}{4} \div 7\frac{2}{3}$.

$$\text{Height} = \frac{51\frac{3}{4}}{7\frac{2}{3}} = \frac{207}{4} \div \frac{23}{3}$$

$$= \frac{207}{4} \times \frac{3}{23} = 6\frac{3}{4} \text{ ft.}$$

The same methods are applicable when letters in addition to numbers occur in the given expressions.

EXAMPLE 5. *Divide* $\frac{8}{ab}$ *by* $\frac{4}{a}$.

$$\frac{8}{ab} \div \frac{4}{a} = \frac{8}{ab} \times \frac{a}{4} = \frac{2}{b}.$$

EXAMPLE 6. *Divide* $\frac{ab}{bc}$ *by* $\frac{2a}{3cd}$.

$$\frac{ab}{bc} \div \frac{2a}{3cd} = \frac{ab}{bc} \times \frac{3cd}{2a} = \frac{3d}{2}.$$

Conversion of Fractions.—The conversion of a vulgar into a decimal fraction may be effected by dividing the numerator by the denominator. As many ciphers as are necessary to carry on the division are added: or, in many cases, and more especially in the simpler vulgar fractions, the conversion is effected by multiplying or dividing numerator and denominator by a suitable number, so that the denominator becomes ten or some multiple of ten.

It is advisable to commit to memory the equivalent decimal fractions of some of the more commonly occurring vulgar fractions.

Thus $\frac{3}{4} = \frac{75}{100} = 0{\cdot}75$, $\frac{1}{2} = \frac{5}{10} = 0{\cdot}5$, $\frac{1}{4} = \frac{25}{100} = 0{\cdot}25$,
$\frac{1}{8} = \frac{125}{1000} = 0{\cdot}125$, $\frac{1}{16} = \frac{625}{10000} = 0{\cdot}0625$, etc.

In some cases the figures in the quotient do not stop, thus $\frac{1}{3} = 0{\cdot}33\ldots.$

The result of the division may be shown by as many threes as we care to write. The notation $0{\cdot}\dot{3}$ is used to denote this unending row. Similarly $\frac{2}{3} = 0{\cdot}666\ldots = 0{\cdot}\dot{6}$. These are known as **recurring, repeating,** or sometimes as **circulating** decimals.

In each of these, and in all similar cases, the equivalent vulgar fractions are obtained by writing 9 instead of 10 in the denominator, thus

$$0{\cdot}\dot{3} = \tfrac{3}{9} = \tfrac{1}{3}, \quad 0{\cdot}\dot{6} = \tfrac{6}{9} = \tfrac{2}{3}, \quad 0{\cdot}\dot{6}\dot{3} = \tfrac{63}{99} = \tfrac{7}{11}.$$

A fraction such as $0{\cdot}1\dot{2}\dot{3}$ is called a **mixed recurring decimal**, and the equivalent fraction is obtained by *subtracting the non-recurring figures from all the figures to obtain the numerator, and writing as many nines as there are recurring figures, followed by as many ciphers as there are non-recurring figures for the denominator.*

Thus
$$0{\cdot}1\dot{2}\dot{3} = \frac{123 - 1}{990} = \frac{122}{990} = \frac{61}{495},$$

$$0{\cdot}21\dot{0}\dot{4} = \frac{2104 - 21}{9900} = \frac{2083}{9900}.$$

EXERCISES XI.

Multiply together

1. $\frac{3}{4}$, $\frac{1}{2}$.
2. $\frac{7}{8}$, $\frac{3}{8}$.
3. $\frac{5}{16}$, $\frac{4}{7}$, $\frac{5}{8}$.
4. $\frac{3}{64}$, $\frac{8}{9}$, $\frac{3}{8}$, $\frac{5}{16}$.
5. $1\frac{3}{4}$, $1\frac{2}{13}$, $2\frac{1}{6}$, $\frac{6}{7}$.
6. $2\frac{21}{32}$, $\frac{14}{51}$, $1\frac{13}{35}$.
7. $2\frac{7}{16}$, $\frac{15}{64}$, $4\frac{4}{7}$, $\frac{14}{65}$.
8. $1\frac{5}{16}$, $\frac{3}{14}$, $3\frac{5}{9}$.
9. $\frac{5}{7}$, $5\frac{2}{5}$, $4\frac{2}{3}$.
10. $\frac{18}{59}$, $1\frac{17}{42}$, $1\frac{5}{9}$.
11. $\frac{3}{10} \times 4\frac{2}{3} \times 2\frac{3}{16}$.
12. $17\frac{11}{17} \times 1\frac{14}{37} \times 4\frac{14}{15} \times \frac{1}{200}$.

Divide

13. $\frac{1}{3}$ by $\frac{1}{4}$.
14. $\frac{1}{2}$ by $\frac{5}{8}$.
15. $\frac{5}{8}$ by $\frac{3}{8}$.
16. $8\frac{11}{56}$ by $2\frac{13}{70}$.
17. $\frac{13}{96}$ by $\frac{65}{84}$.
18. $8\frac{29}{72}$ by $3\frac{61}{108}$.
19. $\frac{77}{156}$ by $15\frac{31}{99}$.
20. $8\frac{17}{32}$ by $1\frac{43}{48}$.
21. $3\frac{7}{22}$ by $1\frac{7}{66}$.
22. $2\frac{3}{14}$ by $8\frac{6}{7}$.
23. Express as vulgar fractions: $0{\cdot}1\dot{2}\dot{3}$, $0{\cdot}1\dot{4}2857\dot{}$, $0{\cdot}32\dot{6}5\dot{7}$.

Multiply together

24. $\frac{ab}{bc} \times \frac{cd}{ad}$.

25. $\frac{15x^3z}{36y^2z^2} \times \frac{9x^5z^4}{30xy^2z^2} \times \frac{y^2}{x^7}$.

26. $\frac{1}{x+3} \times \frac{(x+3)^2}{x+4}$.

27. Divide $207a^5b^7c^4$ by $23a^3b^2c^2$.

Simplification of Fractional Expressions.—When two or more fractions are connected by the symbols of addition, subtraction, multiplication, or division, the process of simplification simply consists in performing the various processes indicated in order, and presenting the result as a fraction in its lowest terms.

EXAMPLE 1. *Simplify*

$$(2\tfrac{1}{5}+3\tfrac{3}{4}) \times (9\tfrac{1}{2}-7\tfrac{4}{7}) \times (\tfrac{2}{3}-\tfrac{1}{1\,5}-\tfrac{2}{4\,5}).$$

$$\left(\frac{11}{5}+\frac{15}{4}\right) \times \left(\frac{19}{2}-\frac{53}{7}\right) \times \left(\frac{30-3}{45}-\frac{2}{\quad}\right)$$

$$= \left(\frac{44+75}{20}\right) \times \left(\frac{133-106}{14}\right) \times \left(\frac{25}{45}\right)$$

$$= \frac{119}{20} \times \frac{27}{14} \times \frac{25}{45} = 6\tfrac{3}{8}.$$

EXAMPLE 2. *Two taps when running can separately fill a bath in 4 hours and 6 hours respectively. Find the time required when both are running.*

The first tap will fill $\frac{1}{4}$ of the bath in 1 hour and the second $\frac{1}{6}$ in the same time.

Hence, when both are open $\frac{1}{4}+\frac{1}{6}=\frac{5}{1\,2}$ is filled in 1 hour, and time required is $\frac{1\,2}{5}=2\frac{2}{5}$ hours.

The result obtained may be easily verified. Thus, the first tap in $2\frac{2}{5}$ hours will fill $\frac{1}{4} \times \frac{1\,2}{5}=\frac{3}{5}$ of the bath, and the second tap will fill $\frac{1}{6} \times \frac{1\,2}{5}=\frac{2}{5}$. Hence $\frac{3}{5}+\frac{2}{5}=1$.

An extension of the previous problem may be made as follows:

EXAMPLE 3. *Two taps can separately fill a bath in 4 and 6 hours respectively, and a waste pipe can empty it in 5 hours. In what time will the bath be filled if all three are opened together?*

In one hour the two taps will fill $\frac{1}{4}+\frac{1}{6}=\frac{5}{1\,2}$ of the bath; in the same time the waste pipe will empty $\frac{1}{5}$. Hence, in one hour,

$$\frac{5}{12}-\frac{1}{5}=\frac{25-12}{60} \text{ or } \frac{13}{60} \text{ of the bath will be filled;}$$

$$\therefore \text{ time required is } \frac{60}{13}=4\tfrac{8}{1\,3} \text{ hours.}$$

MISCELLANEOUS EXERCISES XII.

Find the value of

1. $\frac{5}{9}$ of $6\frac{3}{4}$ of $\frac{16}{17}$. **2.** $2-(\frac{1}{4}+\frac{1}{5}+\frac{1}{6})$.

3. Simplify $(\frac{5}{6}+7\frac{3}{8}+1\frac{1}{2})\div(\frac{3}{4}-\frac{5}{8})$.

4. Subtract $\frac{3}{5}$ from $1\frac{19}{40}$.

5. Simplify $5\frac{1}{3}$ of $4\frac{1}{2}-3\frac{1}{4}$ of $3\frac{1}{5}$.

6. Divide $3\frac{1}{3}-2\frac{1}{5}$ by $3\frac{1}{3}+2\frac{1}{5}$.

7. Subtract $\frac{3}{7}$ of $\frac{5}{8}$ from $\frac{9}{10}$ of $3\frac{3}{4}$.

8. Subtract $\frac{2}{9}$ of $5\frac{1}{4}$ of $\frac{3}{14}$ of 17 from 30.

9. A shareholder owns $\frac{5}{11}$ of a share; if he sells $\frac{1}{3}$ of it, what fraction of a share will he have left?

10. A man invests £125. 5*s.* in a ship, one-eighth of which is worth £36. 10*s.* 7½*d.* Find what fraction of the value of the ship his share will represent.

11. Seven-elevenths of a field is made into garden allotments, and the remainder is meadow land; the difference between the areas of the allotments and the meadow land is 3 acres. Find the size of the field.

12. A bankrupt's debts amount to £600 and his assets to £410. What amount can be paid in the pound and what will a creditor receive to whom he owes £125?

13. *A* contributes £500 to a business, *B* £300, and *C* £200. If the total profits are £80, what amount of the profit is due to each?

14. A cistern can be filled by one pipe in 25 minutes and emptied by another in three-quarters of an hour. If both pipes are opened together, in what time will the cistern be full?

15. Two pipes together can fill a cistern in one hour. One of them alone fills it in three hours; how long will it take the other to fill it?

16. A hot-water tap can fill a bath in 5 minutes and a cold-water tap can fill it in 3 minutes. If the waste-water pipe can empty it in 7½ minutes, in what time will the bath be filled when all are opened together?

Metric Measures of Area.

1 square metre = 100 square decimetres.
1 square decimetre = 100 square centimetres.
1 square centimetre = 100 square millimetres.

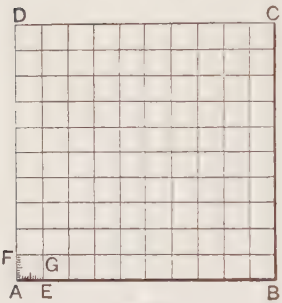


Fig. 36.

Example 1. *Show graphically the number of square decimetres in 1 square metre and the number of square centimetres in a square decimetre.*

Mark off on squared paper ten of the larger divisions (Fig. 36) from A to B, and a similar number vertically from A to D. Then the lines on the squared paper forming the square $ABCD$ are seen to enclose 100 equal squares each 1 decimetre side.

The lines on the squared paper also show that a square decimetre $AEFG$ contains 100 square centimetres.

Conversion Table.

1 sq. in. = 6·45 sq. cm.
1 sq. ft. = 929 sq. cm. = 0·0929 metre.
1 sq. yd. = 8361·13 sq. cm.
1 acre = 4046·7 sq. metres.
1 sq. mile = 2·59 sq. km.
1 sq. cm. = 0·155 sq. in.
1 sq. metre = 10·764 sq. ft. = 1·196 sq. yd.
1 sq. kilom. = 0·386 sq. mile.

Example 2. *To show graphically the number of square inches in a given number of square centimetres. Find* (i) *the number of square centimetres in* 4 *sq. in.*, (ii) *the number of square inches in* 12·9 *sq. cm.*

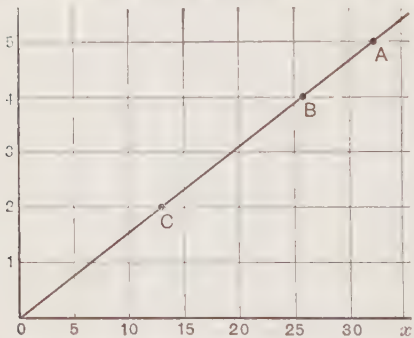


Fig. 37.—Conversion of square inches to square centimetres.

1 sq. in. = 6·45 sq. cm.
5 sq. in. = 32·25 sq. cm.

Mark off on squared paper along Ox five equal divisions each denoting 5 sq. cm., and on the vertical scale let each division equal 1 sq. in. The point A denotes 32·25 sq. cm. and 5 sq. in. Join A to O, then any intermediate value can be read off.

(i) At B the number of square centimetres corresponding to 4 sq. in. is seen to be 25·8.

(ii) At C denoting 12·9 sq. cm. the number of square inches is 2.

EXERCISES XIII.

1. Find the number of square metres in 1000 square yards.
2. Find which is greater, 10 sq. m. or 12 sq. yds., and express the difference between these areas as a decimal of a square metre.
3. How many square centimetres are there in 6 square feet?
4. If 1927 hectares of land are sold for 10,100,000 francs, what is the price per acre? [1 ac. = 0·40467 hectare, £1 = 25$\frac{1}{4}$ francs.]
5. Find the number of square feet in 220 sq. metres.
6. Find the number of square metres in 220 sq. feet.
7. Find the number of square inches in 250 sq. cm.
8. What is the number of square centimetres in 390 sq. in.?

Multiplication of Decimals.—The area of a rectangle is obtained by multiplying the number of units in the length by the number in the width (p. 39).

EXAMPLE 1. *The length of a rectangle is* 7·3 *cm. and its width is* 3 *cm. Find its area.*

Area = 7·3 × 3 sq. cm.

The product of 73 and 3 is found by multiplication to be 219. As there is one decimal place it is necessary to mark off one decimal place in the product and the result is 21·9.

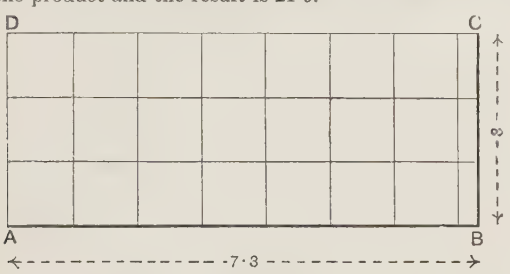


FIG. 38.

Draw a rectangle *ABCD*, Fig. 38, and through the points of division draw lines parallel to the sides; then 7 × 3 = 21 squares are enclosed together with three rectangles 3 × 0·3 = 0·9. Hence area of figure *ABCD* = 21·9.

EXAMPLE 2. *The length of a rectangle is* 7$\frac{5}{8}$ *in. and its width is* 3$\frac{3}{8}$ *in. Find its area.*

$7\frac{5}{8} = 7{\cdot}625$ and $3\frac{3}{8} = 3{\cdot}375$;

$\therefore\ 7\frac{5}{8} \times 3\frac{3}{8} = 7{\cdot}625 \times 3{\cdot}375.$

The product of 7625 and 3375 is found to be 25734375. As there are three decimal places in each of the two given numbers it is necessary to mark off $3+3=6$ decimal places in the product ;

$$\therefore\ 7{\cdot}625\times 3{\cdot}375=25{\cdot}734375.$$

Example 3. *Find the area of the top of a table if the length is* 8 *ft.* 3 *in. and the width* 4 *ft.* 6 *in.*

$$8 \text{ ft. } 3 \text{ in.}=8\tfrac{1}{4} \text{ ft.}=8{\cdot}25 \text{ ft., and } 4 \text{ ft. } 6 \text{ in.}=4{\cdot}5 \text{ ft. ;}$$

$$\therefore \text{ area}=8{\cdot}25\times 4{\cdot}5=37{\cdot}125 \text{ sq. ft.}$$

Example 4. *Find the product of* 0·0024 *and* 0·038 *to two significant figures.*

Here the product of 24 and 38 is 912, there are $4+3=7$ decimal places to be marked off in the product, hence four ciphers must be placed before the first significant figure and the result is 0·0000912.

Significant Figures.—What is usually wanted in any practical application of arithmetical rules is the product to a **given number of significant figures**, or to a given number of decimal places. The rule adopted is that since any rejected figure greater than 5 is nearer 10 than 0, in rejecting such a figure, 1 is added to the preceding digit : when the rejected figure is less than 5, the preceding figure is left unaltered. Thus, in Example 2, the result, true to four decimal places, would be 25·7344 and to three decimal places 25·734.

Similarly, in Example 4, the result true to two significant figures is 0·000091.*

Example 5. *The sides of a rectangle are measured and found to be* 2·75 *in. and* 2·53 *in. respectively. Find the area of the rectangle. Assuming that there is a possible error in the measurements and they are* 0·01 *too small, what is the area?*

The area of the rectangle, using the given dimensions, is

$$2{\cdot}75\times 2{\cdot}53=6{\cdot}9575 \text{ sq. in.}$$

But if these dimensions are each too small by the one-hundredth of an inch, then the sides should be 2·76 and 2·54 in. respectively, and the area is $2{\cdot}76\times 2{\cdot}54=7{\cdot}0104$ sq. in.

Thus, in dealing with the product of measured lengths, it is inaccurate to assume that the result is correct to four decimal places. Even assuming the numbers given to be accurate, since no result can be more accurate than the data from which it is obtained, the preceding result should be

* The difference between the two phrases should be carefully noted ; thus, in Ex. 4, the first *three significant figures* are 912, the first *seven decimal places* are 0·0000912.

6·96 sq. in.; or, assuming that there may be a slight possible error in the dimensions, then the area can only be given as 7 sq. in.

In these cases an approximate answer is usually sufficient and this is readily obtained by **contracted methods.**

Contracted Multiplication.

EXAMPLE 1. *Find the product of* 3·65 *and* 7·62 *correct to two decimal places.*

(i) Ordinary Method.	(ii) Contracted Methods.	(iii)
3·65	3·65	3·65
7·62	7·62	267
730	25·55	2555
2190	2·190	219
2555	7·30	7
27·8130	27·81	27·81

The ordinary, or long, method shown at (i) will not present any difficulty.

In (ii) commence by multiplying the upper line by the left-hand figure 7 and obtain the product 2555. Next multiply by 6, writing the product 2190 one place to the right. Finally, multiply by 2, place the result two places to the right and add to the preceding product. The final result is 27·81; the two remaining figures are not retained.

At (iii) the multiplier is reversed and the product obtained as in the preceding case.

EXAMPLE 2. *The circumference of a circle may be obtained by multiplying the diameter of the circle by* 3·142.

If the diameter of a circle is 27·75 *in., find its circumference.*

$$27{\cdot}75 \times 3{\cdot}142 = 87{\cdot}18.$$

The product is conveniently obtained by the contracted method.

27·75
2413
8325
277
111
5
87·18

Similar methods are applicable when the continued product of three or more numbers is required.

EXAMPLE. 3. *Find the product of* 27·75, 3·142 *and* 25·36.

The product of 27·75 and 3·142 is 87·18 and $87{\cdot}18 \times 25{\cdot}36 = 2210{\cdot}8$.

The product of two or more numbers may be obtained by contracted methods, as indicated in the preceding examples, but when the number of decimal places in the product is specified, the process admits of further simplification.

EXAMPLE 4. *Find the value of* 53·653426 *and* 42·52362 *correct to three decimal places.*

Reverse the multiplier and use the contracted method. The product is 2281·53789.

It will be noticed that more figures are retained than are necessary. To ensure that the last figure in the product is correct it is necessary to retain four decimal places in the product.

Hence, reversing the multiplier and placing the first figure of the multiplier under the fifth decimal place in the multiplicand we shall retain four decimal places in the product and a suitable correction can then be made for the figure rejected.

The process is as follows:

$$\begin{array}{r} 53{\cdot}653426 \\ 2632524 \\ \hline 21461370 \\ 1073068 \\ 268267 \\ 10731 \\ 1609 \\ 321 \\ 11 \\ \hline 2281{\cdot}5377 \end{array}$$

Put the first figure 4 in the multiplier under the fifth decimal place, then as the product 4×6 is 24, the 4 is not written down but the 2 is carried to the next figure, giving $4 \times 2 + 2 = 10$; the 0 is written, and the 1 carried in the usual manner to the next figure. The sum of all the products obtained in this way gives the result 2281·5377, or, correct to three decimal places, is 2281·538.

Similarly, to obtain the product correct to two decimal places, three decimal places are retained in the product and the first figure of the multiplier is written under the fourth in the multiplicand.

$$\begin{array}{r} 53{\cdot}653426 \\ 2632524 \\ \hline 2146137 \\ 107307 \\ 26827 \\ 1073 \\ 161 \\ 32 \\ \hline 2281{\cdot}537 \end{array}$$

Here, as 4×2 is greater than 5, 1 is carried to the next figure, thus $4 \times 4 + 1 = 17$; the 7 is written down, and the 1 carried to the next figure. Similarly, in the second row, as 2×4 is greater than 5, 1 is carried to the next figure, giving $2 \times 3 + 1 = 7$, and so on.

The sum of all these products is 2281·537 and this, correct to two decimal places, is written as 2281·54.

In a similar manner, if the result is to be correct to one decimal, place the first figure of the multiplier under the third figure in the multiplicand thus:

```
  53·653426
2632524
---------
 214614
  10731
   2682
    107
     16
---------
2281·50
=========
```

Here $4 \times 4 = 16$ giving 1 to carry to the next figure; also, as 6 is greater than 5, we say $4 \times 3 + 2 = 14$, hence 4 is written down and the 1 carried to the next figure.

The method will be obvious from the working shown. In this case two figures in the upper line and two in the lower are not used. In the preceding case, one figure in each line is not used.

EXERCISES XIV.

1. 8297·41 × 0·59 correct to three decimal places.
2. 4630·98 × 7·38 correct to four decimal places.
3. 2356·225 × 415·35 correct to four decimal places.
4. 2778·588 × 98·67 correct to three decimal places.
5. 416·8952 × 0·04568 correct to five decimal places.
6. 5·39 × 21·07 and the product by 0·0013 to six decimal places.
7. 39·0437 × 5·40305 to five decimal places.
8. $0{\cdot}3949 \times \frac{5\cdot4}{3\cdot9\cdot5}$ to two significant figures.
9. 0·4375 × 30·16 to two decimal places.
10. 0·0426 × 575.
11. 0·0713 × 470 to two decimal places.
12. 0·000563 × 296000.
13. 36·05 × 98·9.
14. 2890 × 0·0346 to two decimal places.

15. $1{\cdot}17 \times 0{\cdot}0311$ to three significant figures.

16. $34{\cdot}01 \times 0{\cdot}037$ to three decimal places.

17. $30{\cdot}56 \times 4{\cdot}105$ to three decimal places.

18. $23{\cdot}07 \times {\cdot}1354$ to three decimal places.

19. $8{\cdot}102 \times 35{\cdot}14$ to one decimal place.

20. The area of one side of a wall is length × height. Find the area of the side of a wall (i) 12·39 ft. long and 5·024 ft. high; (ii) 18·25 ft. long, 8·75 ft. high.

21. The area of a circle whose radius is 10·25 in. is $3{\cdot}1416 \times (10{\cdot}25)^2$. Find the area correct to two decimal places.

22. The area of a circle whose radius is 33·375 is $3{\cdot}1416 \times (33{\cdot}375)^2$. Find the area correct to one decimal place.

23. The circumference of a circle whose radius is 10·25 in. is $3{\cdot}1416 \times 2 \times 10{\cdot}25$. Find the circumference correct to three decimal places.

24. Find to two decimal places the circumference and area of a circle whose radius is 12·125 in.

Division of Decimals.—When one number is divided by another, the quotient may be obtained either by the ordinary method (p. 4) or by some form of contracted method. The process, when decimals enter into the computation, is the same as for whole numbers. The results obtained may be verified by multiplying the divisor by the quotient; the product, together with the remainder if any, should equal the dividend.

The following example will illustrate the method adopted.

EXAMPLE 1. *Divide* 87·18 *by* 3·142.

$$
\begin{array}{r}
3{\cdot}142 \,)\, 8718 \,(\, 27{\cdot}75 \\
6284 \\
\hline
2434 \\
2199 \\
\hline
235 \\
220 \\
\hline
15 \\
15 \\
\hline
\end{array}
$$

By inspection the quotient is seen to consist of two figures followed by decimals. The first figure in the quotient is 2, and the product $2 \times 3{\cdot}142 = 6284$. By subtracting this result from 8718 we get a remainder of 2434. The next figure in the quotient is 7; multiply the divisor by 7; then $7 \times 2 = 14$, the 4 is not written down but the 1 is

carried to the next figure ; $7 \times 4 = 28$ and 1 carried from the last figure gives 29, the 9 is written down and 2 carried to the next figure in the usual manner ; the remainder is found to be 235. The 2 in the divisor is not used again and may be crossed out, or a dot or cross placed over or under it. Next $7 \times 4 = 28$ and as 8 is greater than 5, $2 + 1 = 3$ is carried to the next figure, thus $7 \times 1 + 3 = 10$, the 0 is written down and $7 \times 3 + 1 = 22$. Subtracting 220 from 235 the difference is 15, and thus the quotient is 27·75.

It will be seen that in the contracted method one figure of the divisor is rejected at each step of the work.

Example 2. *The area of a rectangle is* $37\frac{1}{8}$ *sq. ft. If one side of the rectangle is* 4 *ft.* 9 *in. long, what is the length of the other?*

$37\frac{1}{8} = 37{\cdot}125$ and 4 ft. 9 in. $= 4{\cdot}75$ ft. ;

$\therefore$ length $= 37{\cdot}125 \div 4{\cdot}75 = 7{\cdot}816$ ft.

The result obtained may be verified by multiplying 7·816 by 4·75.

Italian Method.—A contracted method usually referred to as the *Italian Method* is frequently used, in which the only results which are written down are those of the several subtractions.

Example 3. *To divide* 113811·0925 *by* 479·75.

The alternate lines only are written down as follows :

479·75) 113811·0925 (237·23

178610

346859

110342

143925

At each step one line of figures is dispensed with, and the working takes up less room than is the case in the ordinary or long method.

EXERCISES XV.

Divide

1. 1406·373 by 10·8 correct to two decimal places.
2. 5685·4327 by 7·323 to one decimal place.
3. 273410·28 by 3·12 to one decimal place.
4. 732·506 by 0·153.
5. 15497·9552 by 41·6 to three decimal places.

Divide

6. 2·114 by 6·04 to two decimal places.

7. 0·056068 by 8·0327 to three significant figures. What is meant by "significant figures"?

8. 2·332714 by 390·086 to three significant figures.

9. 0·3493296 by 11·416 to two significant figures.

10. 0·32406 by 0·00893 to three decimal places.

11. 0·0165577 by 2·873 to four significant figures.

12. 0·0232 by 0·002873 to three decimal places.

13. 51·2585 by 17·081 to six significant figures.

14. 2101·127 by 150·079 to five decimal places.

15. 2·4008 by 0·070605 to five decimal places.

16. 0·0518207 by 0·00734 to two decimal places.

17. 0·0000001645 by 0·0047 to two significant figures.

18. 0·00918 by 0·018 to two decimal places.

Decimals of Concrete Quantities.—Fractional amounts of money, fractional weights, lengths, times and so on, are readily expressed as decimals. The methods adopted may be seen from the following examples:

EXAMPLE 1. *Express* 6*d.* *as the fraction of a shilling.*

As there are 12*d.* in 1*s.*, the required fraction is $\frac{6}{12} = \frac{1}{2} = 0{\cdot}5$.

EXAMPLE 2. *Express* 6*s.* 8*d.* *as the fraction of one pound.*

$$6s.\ 8d. = 6\tfrac{2}{3}s.$$

The required fraction $= \frac{6\frac{2}{3}}{20} = \frac{20}{60} = \frac{1}{3} = 0{\cdot}333,$

or more simply, $6s.\ 8d. \times 3 = £1;$

$$\therefore\ 6s.\ 8d. = £\tfrac{1}{3}.$$

EXAMPLE 3. *What fraction of* £1 *is* 15*s.*?

$$15s. = £\tfrac{15}{20} = £\tfrac{3}{4} = 0{\cdot}75.$$

The required result may frequently be obtained in several ways.

Thus, $3s.\ 4d. \times 6 = £1;$

$$\therefore\ 3s.\ 4d. = £\tfrac{1}{6},$$

or, $3s.\ 4d. = 3\tfrac{1}{3}s.;$

$$\therefore\ 3\tfrac{1}{3} \div 20 = £\tfrac{1}{6},$$

or, $3s.\ 4d. = 40d.,$

and $\frac{40}{240} = £\frac{1}{6}.$

Weight. EXAMPLE 4. *Express* 2 *cwt.* 16 *lb. as the decimal fraction of a ton.*

2 cwt. 16 lb. = 240 lb., and 1 ton = 2240 lb.;

$\therefore$ required fraction $= \frac{240}{2240} = \frac{3}{28} = 0{\cdot}107$.

EXAMPLE 5. *Express* 3 *lb.* 2 *oz. in lb.*

3 lb. 2 oz. $= 3\frac{1}{8}$ lb. $= 3{\cdot}125$ lb.

EXAMPLE 6. *Find the value in lb. of* 0·304 *of a ton.*

1 ton = 2240 lb.;

$\therefore$ $0{\cdot}304 \times 2240 = 680{\cdot}96$ lb.

Length. EXAMPLE 7. *Express* 220 *yards as a fraction of a mile.*

1 mile = 1760 yds.;

$\therefore$ required fraction $= \frac{220}{1760} = \frac{1}{8} = 0{\cdot}125$.

EXAMPLE 8. *Find the number of feet in* 0·75 *of a mile.*

1 mile = 5280 ft.;

$\therefore$ $0{\cdot}75 \times 5280 = 3960$ ft.

Time. EXAMPLE 9. *Express* 30 *minutes as the fraction of a day.*

30 min. $= \frac{1}{2}$ hour, 1 day = 24 hours;

$\therefore$ required fraction $= \frac{1}{2}$ of $24 = \frac{1}{48}$,

or $$\frac{30}{60 \times 24} = \frac{1}{48} = 0{\cdot}02084.$$

EXAMPLE 10. *Find the number of hours in* 0·035 *of a week.*

1 week $= 7 \times 24$ hours;

$\therefore$ $0{\cdot}035 \times 7 \times 24 = 5{\cdot}88$ hours.

As there are 60 minutes in an hour and 60 seconds in a minute; then 0·88 hour is 52 min. and 48 sec. (p. 225), or 5·88 hours is 5 hr. 52 min. 48 sec.

A given number of shillings may be expressed as a decimal fraction of £1 by calculation, or may be read off by using squared paper.

EXAMPLE. 2*s.* = £0·1, 10*s.* = £0·5, 6*s.* 8*d.* = £0·3.

Select any horizontal line on a piece of squared paper, and take a number of divisions to denote shillings. On a vertical scale let the

divisions denote fractional parts 0·1, 0·2, etc., of £1. At the intersection of the vertical line through 2, and the horizontal line through 0·1 the point *A* (Fig. 39), denoting that 2*s.* is £0·1, is found. Similarly, *B* is the intersection of lines through 6 and 0·3; and finally, *C* the intersection of lines through 10 and 0·5. By drawing a fine line through the three points, the decimal fraction corresponding to a given number of shillings up to 10*s.* can be read off. Thus, 8*s.* is seen to be £0·4, and 6*s.* 6*d.* is £0·325.

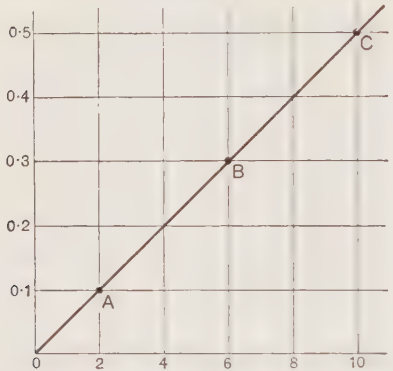


FIG. 39.—Conversion of a number of shillings to decimals of £1.

By marking off 12 equal divisions between the figures 1, 2, 3, ..., denoting shillings, the decimal fraction corresponding to any number of shillings and pence can be obtained.

The conversion of quantities representing weight, length and time, etc., can be effected in a similar manner.

MISCELLANEOUS EXERCISES XVI.

1. Simplify $(\frac{5}{8}+\frac{3}{4}+2{\cdot}46)\div 125$.

2. Find the sum, the difference, the product and the quotient of the two numbers 0·04 and 0·005. Find the sum of the five results.

3. Find the value of $(1-0{\cdot}24+0{\cdot}36+0{\cdot}18)300$.

4. Find the cost of painting one side of a wall $36\frac{3}{4}$ ft. long, and 9 ft. 6 in. high, at 1*s.* $2\frac{1}{2}$*d.* per square yard.

5. Convert into decimals $\frac{1}{4}$, $\frac{1}{2}\frac{6}{5}$, $\frac{4}{5}\frac{3}{0}$, $3\frac{1}{2}$, and simplify

$$\left(\frac{1}{4}+\frac{1}{2}\frac{6}{5}+\frac{4}{5}\frac{3}{0}+3\frac{1}{2}\right)\div 175,$$

the result to be true to two significant figures.

6. Find, to four decimal places, the value of $(\frac{4}{7}+\frac{3}{8})\times(\frac{4}{7}-\frac{3}{8})$.

7. Express $(14{\cdot}4+1{\cdot}44)\div(14{\cdot}4-1{\cdot}44)$ as a vulgar fraction in its simplest form.

8. Find the value of $(18{\cdot}3+21{\cdot}065)\div(25{\cdot}1-17{\cdot}09867)$ to four decimal places.

9. Find, correct to four decimal places, the sum of 0·15, 2·01246, 0·1033 and 82·2 divided by 10·24.

10. Find, correct to four significant figures, the value of 0·049852 divided by 1·3569.

11. Add $\frac{2}{3}\left(3\frac{1}{1}\frac{1}{1} - 2\right)$ to $\frac{3}{4} \times 0{\cdot}0603 \div \frac{5}{4} \times 0{\cdot}00594$.

12. (*a*) Reduce 15*s*. 7½*d*. to the decimal of £5.
(*b*) Find the value of 0·001 of £18. 15*s*.

13. (*a*) Reduce 15*s*. 4½*d*. to the decimal of £1.
(*b*) Find the value of 0·0125 of a ton.

14. (*a*) Reduce 5*s*. 6*d*. to the decimal of £1.
(*b*) Find the value of 0·325 of 1½ tons.

15. (i) Reduce 2·05 of half-a-crown to the fraction of a pound.
(ii) Add together 2·2 of five shillings + 0·22 of ten shillings.

16. (i) Add together $\frac{7}{2\,4}$ of £6. 5*s*. 4*d*. and $\frac{1}{6}$ of £12. 7*s*. 2*d*. ;
(ii) express $\frac{2}{9}$ of their sum as the fraction of £2. 11*s*. 10*d*.

17. In a town of 2000 inhabitants, it is found that 0·3 of the population are men, 0·175 are women and the remainder children. Find the number of men, women and children.

18. One man owns ¼ of a ship, another owns ⅓ of the remainder. If the value of the ship is £6800, find the share of each, and the value of their united shares.

19. To $\left(2{\cdot}5 + 3{\cdot}625 + \frac{4}{9} + \frac{1}{7}\frac{3}{2}\right)$ add $4\frac{1}{5} \times \frac{1}{6}\frac{0}{3}$ and from the sum subtract $2\frac{3}{1\,4} \div 8\frac{6}{7}$.

20. To $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} - 1{\cdot}25 + 3{\cdot}5\right)$ add $\frac{1}{5\,2}$ of $4\frac{1}{3}$ and from the sum subtract $5\frac{1}{8} \div 1\frac{7}{8}$.

Averages.—One of the simplest methods in use to ascertain the **mean** or **average value** of a series of numbers is to add the numbers together and divide the sum of the numbers by the number in the series.

Example 1. *The weights of the eight men forming a boat's crew are* 9 *st*. 8 *lb*., 10 *st*. 6 *lb*., 12 *st*. 4 *lb*., 12 *st*. 2 *lb*., 10 *st*. 4 *lb*., 11 *st*. 3 *lb*., 11 *st*. 8 *lb*. *and* 12 *st*. 7 *lb*. *What is the average weight?* [1 st. = 14 lb.]

Add the given weights together; the sum is 90 st. The average weight is obtained by dividing this sum by the number of men.

$$\therefore\ 90 \div 8 = 11 \text{ st. } 3\tfrac{1}{2} \text{ lb.}$$

It will be noticed that the average weight so obtained does not necessarily give the weight of any man in the crew.

But the average obtained is such that, when taken as many times as there are given quantities, the result is the same as that obtained by adding all the given quantities together. Thus, the total weight of the boat's crew may be found by multiplying the average weight by the number of men.

EXAMPLE 2. *There are 90 persons in a street of 15 houses. What is the average number of persons per house?*

$$\text{Average number} = \frac{90}{15} = 6.$$

It has been learnt that the area of a surface is obtained by multiplying the number of units in its length by the number of units in its width; and similarly the volume of a body is the product of its length, width and thickness. In each case the dimensions are measured quantities, and as such are liable to error. Hence the dimensions in each case are assumed to be the mean or average of two or more readings.

Thus, for example, one observer may measure the two dimensions of a rectangle and find them to be 5·43 and 4·64 in. respectively. Another observer finds them to be 5·42 in. and 4·63 in. The area in the first case is

$$5{\cdot}43 \times 4{\cdot}64 = 25{\cdot}1952 \text{ sq. in.}$$

The area in the second case is

$$5{\cdot}42 \times 4{\cdot}63 = 25{\cdot}0946 \text{ sq. in.}$$

The true value is probably some intermediate number, and a better result than either of those obtained would be found by using the mean of the given dimensions.

Thus $\frac{1}{2}(5{\cdot}43 + 5{\cdot}42) = 5{\cdot}425,$

and $\frac{1}{2}(4{\cdot}64 + 4{\cdot}63) = 4{\cdot}635.$

Hence, the probable value is given by

$$5{\cdot}425 \times 4{\cdot}635 = 25{\cdot}145 \text{ sq. in.}$$

All results obtained by measurement or observation are average or mean results. Thus, if a train starting from one station reaches another station 150 miles from the former in 3 hours, the average speed is given by dividing the space passed over by the time taken on the journey. It is

$$150 \div 3 = 50 \text{ miles per hour.}$$

EXAMPLE 3. *A motor-car is found to travel over a distance of 200 yards in 10 seconds. Find the average speed of the car in miles per hour.*

As the space described is given by the product of the speed and the time, we obtain

$$(\text{average speed}) \times 10 = 200;$$

$$\therefore \text{ average speed} = 20 \text{ yds. per sec.}$$

$$\therefore \text{ speed per hour} = \frac{20 \times 60 \times 60}{1760} = 40\tfrac{10}{11} \text{ miles per hour.}$$

EXERCISES XVII.

1. If the price of a fixed quantity of wheat for five successive years is 55*s*. 4*d*., 66*s*. 8*d*., 71*s*. 3*d*., 44*s*. and 37*s*. 9*d*., what is the average price for the five years?

2. A pedestrian in 7 days walks the following distances in miles, $11\frac{1}{2}$, $16\frac{3}{8}$, $17\frac{2}{3}$, $8\frac{3}{4}$, $10\frac{3}{8}$, $17\frac{2}{3}$ and $7\frac{1}{2}$. Find the average distance walked per day.

3. Forty boards are on the average $9\frac{1}{4}$ inches in width. Find their total width.

4. The readings of a barometer on five consecutive days are 29·6, 30·1, 30·3, 30·4, 29·8 inches respectively. Find the average reading for the five days.

5. The time taken by a train for a journey of 89·6 kilometres is 1 hour 10 minutes. Find its average speed in centimetres per second.

6. A train travels 143·1 miles in 3·82 hours. Find its average rate per hour, and its time over $4\frac{3}{4}$ miles.

7. If the telegraph posts on a railway are placed at intervals of 70 yards, at what rate in miles per hour is a train travelling which passes three of these intervals in 11 seconds?

8. The average length of 10 sticks is 2 ft. $10\frac{1}{2}$ in.; one is $27\frac{1}{2}$ in. long, another $37\frac{1}{2}$ in. long. Find the average length of the remaining eight.

9. In an eight-oared boat the weight of the whole crew was 90 st. Find the average weight of each oarsman, and the weight of the coxswain which is 8/100 of the weight of the whole crew.

10. The average weight of 5 men is 168 lb.; two of them weigh 140 lb. and 190 lb. respectively. What is the average weight of the remaining three?

11. *A* does a piece of work in 3 days, *B* in 4 and *C* in 6, while a day's work of *D* is the average of the other three. How many days will *D* require to do the work?

12. If a train goes a distance of s miles in x hours, what distance in feet does it move in one second?

Verify and find the distance in feet when $s = 60$ miles, $x = 2$ hours.

CHAPTER III.

Measurement of Volume. Volumes of Liquids. Involution. Evolution.

Measurement of Volume.—All solid objects with which we are familiar have the three dimensions of length, breadth and thickness, or depth. In some cases the thickness may be a very small length, such as in a sheet of lead, tin, or paper, but the volume in all cases is the product of the three dimensions, length, breadth and thickness. As in the case of an area, the volume of a body has no reference to its **shape**; thus, the three dimensions of a cone, cylinder, sphere or prism of equal volume, would not be alike.

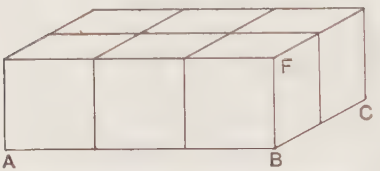


Fig. 40. Volume of a rectangular prism.

A familiar example is furnished by a common brick, or a block of wood, or other object of rectangular form, as in Fig. 40. If the length AB is 3 units, the breadth BC 2 units, and the thickness or height BF is 1 unit, then the area of the base is 6 square units (p. 39).

This fact can be demonstrated by dividing the edge AB into three, and the edge BC into two separate units, then the base is divided into six equal squares, each of one unit side, by drawing lines parallel to BC and AB.

Similarly, by dividing the upper face into the same number of equal parts and drawing vertical lines from the points of

intersection of the parallel lines, the solid is divided into six equal blocks; the edges of the base and the height of each block are of equal length.

Each such block is called a **cube**, and its volume is spoken of as a **cubic unit**.

As a length is measured by the number of times it contains the unit length, and an area by the number of units of area it contains, so the volume of a body is measured by the number of cubes in it, each of unit length of edge.

The length of the edge of the unit cube may be any unit of length, such as centimetre, decimetre, inch, foot and so on; the volume in each of these cases is said to be a cubic centimetre, cubic decimetre, cubic inch and cubic foot respectively.

In Fig. 40, if the edge $AB = 3$ in., $BC = 2$ in. and $BF = 1$ in., then the volume of the solid is $3 \times 2 \times 1 = 6$ cubic inches, usually abbreviated into 6 cub. in.

It is important to notice that the construction shown in Fig. 40 provides a rule for determining the volume of a **rectangular prism**, *i.e.* a solid in which the faces are at right angles to one another, and this is the product of its three dimensions expressed in the same units

Or, as the product of length and breadth gives the area of the base (p. 39) the volume of the solid may be expressed as the product of the **area of the base** and **height**, or area of base × altitude.

Cube.—When the three perpendicular edges of a solid are all equal in length, the solid, as has been stated, is called a **cube**. Thus, if each edge of such a solid represents 1 yard, or 3 feet, the area of the base would be $3 \times 3 = 9$ sq. ft. and the volume $3 \times 3 \times 3 = 27$ cub. ft. Or, 1 cubic yard = 27 cub. ft.

If the edge of a cube is 1 foot, or 12 inches, then the area of the base would be $12 \times 12 = 144$ sq. in. and the volume $12 \times 12 \times 12 = 1728$ cub. in.

Or, 1 cubic foot = 1728 cub. in.

A rectangular prism, length 4 cm., breadth 2 cm. and height 3 cm., could be built up by placing eight unit cubes in two

rows and placing two similar layers of unit cubes on the top. In this manner we obtain a rectangular prism of 24 c.c. volume, which may be obtained by multiplying the area of the base by the height.

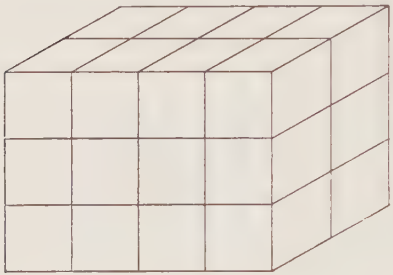


Fig. 41.

If the edge of a cube is 1 metre, then, as 1 metre = 10 decimetres, area of base is $10 \times 10 = 100$ sq. dcm. and volume = 100×10 cubic decimetres (c. dcm.).

Similarly, 1 cubic decimetre = 1000 c.c. and 1 cubic centimetre = 1000 c.mm.

Volumes of Liquids.—The volume of a comparatively small quantity of liquid may be obtained by means of vessels of various kinds known as **Measures.**

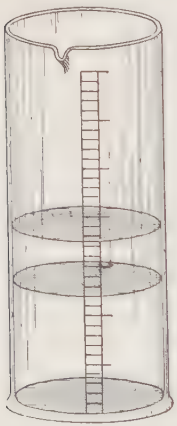


Fig. 42.—Measuring cylinder.

The volume of a small object, or of a quantity of liquid, may be found by using a glass cylinder partly full of water (Fig. 42) furnished with a series of graduations showing the volume of liquid in the vessel. If a small object or a quantity of liquid be put into the cylinder the volume can be ascertained by the difference of the two readings on the scale.

Thus, if the reading showed 200 c.c. before the insertion of the body and 250 c.c. after its insertion, the volume of the body would be

$$250 - 200 = 50 \text{ c.c.}$$

The Imperial Standard of Capacity, or volume contained by a vessel such

as the cylinder referred to above, in the British system, is the volume occupied by a **gallon of water** which **weighs 10 lb.** A gallon is divided into 4 quarts and a quart into 2 pints. The volume of a gallon of water is 0·1604 cub. ft.

One cubic foot of water weighs 62·3 lb.; consequently, the weight of a cubic inch of water is given by

$$62{\cdot}3 \div 1728 = 0{\cdot}036 \text{ lb.}$$

The weight of a given volume of water varies with its temperature. It is usual, therefore, to say that the weight of a cubic foot of water is 62·3 lb. at 62° F.

The unit of volume in the metric system is the cubic metre. For many purposes the smaller units are more convenient, and the unit volume may be the **cubic decimetre** or **cubic centimetre.**

The cubic decimetre is the volume of a cube having its edge 1 decimetre, or 10 centimetres long. The cubic decimetre is called a **litre** and evidently contains

$$10 \times 10 \times 10 = 1000 \text{ c.c. (p. 64).}$$

The conversion from one set of units to another has frequently to be made, and the following relations will be of use for this purpose:

CONVERSION TABLE.

1 cub. inch	= 16·387 cub. centimetres (c.c.).
1 cub. foot	= 28316 c.c.
1 cub. yard	= 764535 c.c. = 0·7646 cub. m.
1 pint	= 567·63 c.c.
1 gallon	= 4541 c.c. = 4·541 litres.
1 cub. metre	= 1·308 cub. yd.
1 cub. decimetre	= 0·0424 cub. ft.
1 cub. centimetre	= 0·061 cub. in.
1 litre	= 61·027 cub. in. = 1·76 pints = 0·22 gall.

EXAMPLE 1. *Show graphically the number of litres in a given number of pints. Use the diagram to find* (i) *the number of pints in 4 litres,* (ii) *the number of litres in 4·55 pints.*

1 litre = 1·76 pints.

5 litres = 8·8 pints.

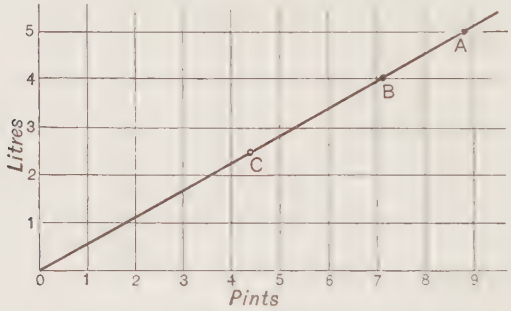


FIG. 43.—Conversion of litres to pints.

Mark the larger squares on squared paper (Fig. 43) horizontally to represent pints, and those in a vertical direction to denote litres.

The point A, denoting 8·8 pints and 5 litres, is joined to the origin O, at which point we may say no pints equal no litres.

(i) The point B clearly corresponds to 4 litres, and a perpendicular on to the scale of pints gives the equivalent of 4 litres to be 7·04 pints.

(ii) The point C, denoting 4·4 pints, and its equivalent in litres can be read off as 2·5 litres.

EXERCISES XVIII.

1. Find the number of cubic inches in a cubic foot and in a cubic yard.

2. How many cubic feet are there in a cubic yard?

3. Find the number of cubic decimetres in a cubic metre.

4. Find the number of cubic centimetres in a cubic decimetre and in a cubic metre.

5. How many cubic millimetres are there in a cubic centimetre?

6. Find the number of gallons in 50 litres, also in a kilolitre.

7. What is the number of litres in 100 pints?

8. Find the number of litres in 10 gallons 3 quarts 1 pint.

9. Given that a gallon measures 0·1604 cubic foot, and that a litre may be represented by a cube, edge 3·937 inches, find the number of pints in a litre.

10. Find the number of cubic feet in 7·3 cubic metres.

11. How many cubic feet are there in 5·7 cubic metres?

12. Find the number of cubic metres in 9·8 cubic feet.

13. How many gallons are there in 8·6 litres?

Involution.—The area of a square is the product of the two numbers representing the length of its sides. Since these numbers are equal, their product has been given a special name called the **square** of the number (p. 8).

Thus, 2×2 or $2^2 = 4$; $0{\cdot}1 \times 0{\cdot}1$ or $0{\cdot}1^2 = 0{\cdot}01$.

If the edge of a cube is 3 ft., its volume is $3 \times 3 \times 3$ or 3^3, read as **3 cubed** or **3 to the power 3.**

The process may be extended thus:

$$2 \times 2 \times 2 = 2^3 = 8, \text{ and } (0{\cdot}1)^3 = 0{\cdot}001; \quad 2^4 = 16, \text{ etc.}$$

The small figure above and to the right of the number is called its **index.**

If a denotes a given length, then the area of a square having that length as a side is a^2, and the volume of a cube having that length as edge is a^3. The relations may be expressed as follows:

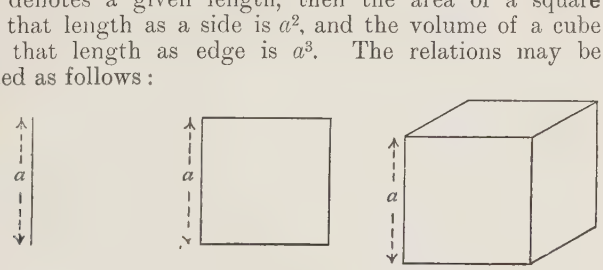


FIG. 44.

a or	a squared or	a cubed, or
a to the first power.	a to the second power.	a to the third power.

The process of obtaining the powers of a number is called **involution.**

The squares and cubes of the numbers 1 to 12 are of frequent occurrence; they should be obtained by

multiplication and the results verified by means of the following table:

Number	1	2	3	4	5	6	7	8	9	10	11	12
Square	1	4	9	16	25	36	49	64	81	100	121	144
Cube	1	8	27	64	125	216	343	512	729	1000	1331	1728

It is advisable to remember the squares of all the numbers up to 12 and if possible up to 20.

Number	13	14	15	16	17	18	19	20
Square	169	196	225	256	289	324	361	400

The squares and cubes of larger numbers and also of numbers involving fractions may be obtained by continued multiplication.

EXAMPLE 1. *Find the area of a square* (i) 2·5 *in.*, (ii) 26·73 *in. side.*

(i) Area $= 2{\cdot}5 \times 2{\cdot}5 = 6{\cdot}25$ sq. in.

(ii) Area $= 26{\cdot}73 \times 26{\cdot}73 = 714{\cdot}49$ sq. in.

The results may be found by arithmetical methods, but these, together with more difficult cases, are readily found by logarithms (p. 109).

Evolution.—Evolution is the converse of involution: by its means we can find the original number, or the **root**, as it is called, when we know the square, cube, or higher power of the number.

The root of a number may be denoted by prefixing the sign $\sqrt{\ }$, called the **radical sign**, together with a number called **the index of the root.**

Thus, $\sqrt[3]{8}$ is read as *the cube root of 8*,

$\sqrt[4]{16}$ as the *fourth root of 16.*

The square root of 16 may be written as $\sqrt[2]{16}$, but in the case of square root it is usual to omit the index of the root, and the square root of 16 is written in the simpler form $\sqrt{16}$.

The square root of a number is, then, one of the two equal factors into which the number can be split. Thus $4 \times 4 = 16$, and $\sqrt{16} = 4$.

The cube root of a number is one of the three equal factors into which the number can be resolved.

Thus, $3 \times 3 \times 3 = 3^3 = 27$; and $\sqrt[3]{27} = 3$.

Similarly, the fourth root of a number is one of the four equal factors of the number.

Thus, $2 \times 2 \times 2 \times 2 = 16$; and $\sqrt[4]{16} = 2$.

Square Root.—When a given number can be written in factors, its square or other root can often be determined by inspection.

Thus, $144 = 12 \times 12$ and $\sqrt{144} = 12$.

$169 = 13 \times 13$; $\therefore \sqrt{169} = 13$.

$125 = 5 \times 5 \times 5$; $\therefore \sqrt[3]{125} = 5$.

EXAMPLE 1. *Find the fourth root of* 50625.

$50625 = 5^4 \times 3^4$; $\therefore \sqrt[4]{50625} = 5 \times 3 = 15$.

EXAMPLE 2. *Find the fifth root of* 3888.

$3888 = 16 \times 243 = 2^5 \times 3^5$;

$\therefore \sqrt[5]{3888} = 2 \times 3 = 6$.

When the square root of a number consisting of several figures—the factors of which are not easily seen—is required, it is necessary to proceed in another way, and to consider the square of an expression consisting of two terms, as in the following example:

73^2	$(70+3)^2$	$(a+b)^2$
73	$70+3$	$a+b$
73	$70+3$	$a+b$
219	$70^2+3\times70$	a^2+ab
511	$3\times70+3^2$	$ab+b^2$
5329	$70^2+2(3\times70)+3^2$	$a^2+2(ab)+b^2$
	$=70^2+(2\times70+3)3$	$=a^2+(2a+b)b$

Similarly, by multiplication we find that

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc,$$

or, the square of an expression of several parts consists of the square of each part together with twice the product of each two parts.

From this we find that when the root of a given number consists of two figures, and the square of one part has been subtracted, the remaining part of the root may be found by using twice the part already found as a trial divisor, in the manner indicated in the following example :

EXAMPLE 1. *Find the square root of* 5329.

As $70^2 = 4900$ and $80^2 = 6400$, the root required evidently lies between 70 and 80.

$$\begin{array}{rl} & 5329\ (70+3=73 \\ & 4900 \\ \hline 140+3=143 & 429 \\ & 3 \\ \hline & 429 \quad 429 \end{array}$$

Subtracting 4900 from 5329 the remainder is 429.

Put twice 70 or 140 on the left as a trial divisor : it will divide 3 times into 429, and by adding 3 we obtain 143 : and this number multiplied by 3 gives 429 ; as there is no remainder the required square root is 73.

The usual practical method is as follows :

$$\begin{array}{r} \dot{5}3\dot{2}\dot{9}\ (73 \\ 49 \\ \hline 143\ ) 429 \\ 429 \end{array}$$

Commence by placing a dot, or point, over the units figure and on each alternate figure to the left. Find the number whose square is nearest to 53 : in this example it is 7. Put this number to the right of the given number and subtract its square, 49, from 53. The remainder is 4. Bring down the next two figures, obtaining 429. Place twice 7 as a trial divisor on the left : it is seen to divide 3 times into 42. Put the 3 with the 14 on the left and also with the 7 on the right ; then multiply 143 by 3 and the product is 429. Hence, the root required is 73.

EXAMPLE 2. *Find the square root of* 541696.

$$\begin{array}{rl} & \dot{5}4\dot{1}6\dot{9}\dot{6}\ (736 \\ & 49 \\ \hline 143 & 516 \\ 3 & \\ \hline 429 & 429 \\ \hline 1466 & 8796 \\ 6 & \\ \hline 8796 & 8796 \end{array}$$

EXAMPLE 3. *Find the square root of* 6177·96.

$$
\begin{array}{r}
\dot{6}\dot{1}\dot{7}\dot{7}\cdot\dot{9}\dot{6} \;(\; 78\cdot6 \\
49 \\
\hline
148\;)\;1277 \\
1184 \\
\hline
1566\;)\;9396 \\
9396 \\
\hline
\end{array}
$$

Commence by "pointing" the units figure and every alternate figure to the left and to the right as shown. The number whose square is nearest to 61 is 7, place 7 on the right and its square, 49, under 61. Subtract and bring down the next two figures, thus obtaining 1277. Using twice 7 as a trial divisor, it is found to divide 8 times into 127; place this number, with the 7, on the right and with the 14 on the left, so obtaining 148. Multiply 148 by 8, and subtract the product, 1184, from 1277, obtaining a remainder 93. Bring down the next two figures. The next trial divisor will be 2×78 or 156. Using this number as a divisor, it is seen to divide 6 times into 939. Hence, put 6 on the right, with the number 156, giving 1566; this number multiplied by 6 leaves no remainder when subtracted from 9396. Hence the root required is 78·6. It will be seen that instead of going back to the square root and doubling the 78, it is only necessary to double the last figure 8 of the 78.

The point or dot is in all cases placed on the units figure and on every alternate figure to the left and to the right of the units figure.

EXAMPLE 4. *The area of a square is* 1428·84 *sq. ft. What is the length of its side?*

$$
\begin{array}{r}
\dot{1}\dot{4}\dot{2}\dot{8}\cdot\dot{8}\dot{4} \;(\; 37\cdot8 \\
9 \\
\hline
67\;)\;528 \\
469 \\
\hline
748\;)\;5984 \\
5984 \\
\hline
\end{array}
$$

Length of side is 37·8 ft.

The area of a circle may be found from the formula $3{\cdot}142 \times r^2$, where r denotes the radius of the circle.

EXAMPLE 5. *If the area of a circle is* 1·6744 *sq. in., find its radius.*

$$r^2 = 1{\cdot}6744 \div 3{\cdot}142 = 0{\cdot}5329.$$

$$\therefore\ r = \sqrt{0{\cdot}5329} = 0{\cdot}73 \text{ in.}$$

Graphical Method.—A graphical method based on Euclid, Book II. Prop. 14, may be used to find the square root of a given number.

EXAMPLE 6. *Find the square root of 5.*

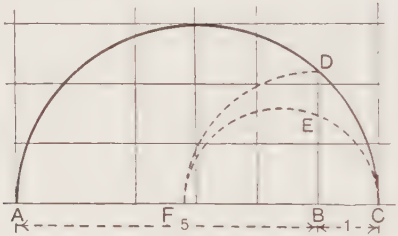


FIG. 45.—Evolution.

Mark off a length AB on squared paper to represent 5 units. From B to C mark off an additional unit. Describe a semicircle on AC, and at B draw a line perpendicular to AB meeting the semicircle in D. Then $BD = \sqrt{5} = 2{\cdot}237$.

The square root of BD may be found in a similar manner. Thus, with B as centre and radius BD, make $BF = BD$, and on FC describe a semicircle. Then, BE, the point of intersection of the perpendicular at B, is the square root required;

$$\therefore \quad BE = \sqrt[4]{5} = 1{\cdot}495.$$

By repeating the process, the square root of BE or $\sqrt[8]{5}$ may be found.

EXAMPLE 7. *The length of the edge of a cube is 6·9 in., find its volume.*

$$\text{Volume} = (6{\cdot}9)^3 = 328{\cdot}5 \text{ cub. in.}$$

To obtain the length of an edge, given the volume, it is necessary to extract the cube root.

EXAMPLE 8. *The volume of a cube is 328·5 cub. in. What is the length of an edge?*

$$\text{Length of edge} = \sqrt[3]{328{\cdot}5} = 6{\cdot}9 \text{ in.}$$

Questions dealing with cubes and cube roots are easily dealt with by logarithms (p. 110).

Root of a Fraction.—A vulgar fraction may be converted into a decimal fraction and the square root found. Or, we may convert the denominator into a complete square

by multiplying the numerator by the same number. It is then only necessary to obtain one root.

EXAMPLE 9. *Find the square roots of* (i) $\frac{3}{8}$, (ii) $\frac{7}{4}$, (iii) $\frac{5}{3}$.

$$\text{(i)}\quad \sqrt{\frac{3}{8}}=\sqrt{\frac{6}{16}}=\frac{1}{4}\sqrt{6}.$$

$$\text{(ii)}\quad \sqrt{\frac{7}{4}}=\sqrt{\frac{28}{16}}=\frac{1}{4}\sqrt{28}.$$

$$\text{(iii)}\quad \sqrt{\frac{5}{3}}=\sqrt{\frac{15}{9}}=\frac{\sqrt{15}}{3}.$$

EXAMPLE 10. *Plot the curve* $y=x^2$. .....(i)

Read off the values $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ *and* $\sqrt{10}$.

When $x=2$, then from (i) $y=4$. Other values may be found and tabulated.

Values of x	0	1	2	3	4	5	6
y	0	1	4	9	16	25	36

Plot the values and draw a fair curve through the plotted points. The required values can be read off from the curve. Thus, $\sqrt{10}$ is seen at F, Fig. 46, to be 3·162.

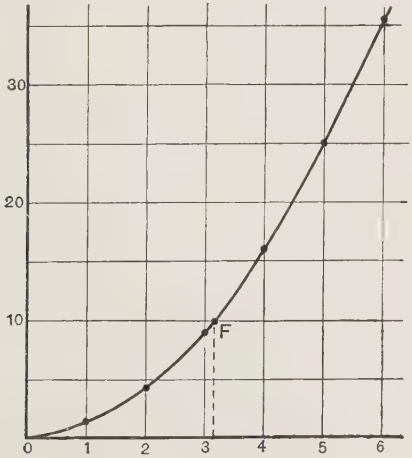


FIG. 46.—Graph of $y=x^2$.

EXERCISES XIX.

Find the square root of

1. 625. 2. 27489049.

3. Find the square and cube of 3·057 in each case to three decimal places.

4. Divide the cube of 0·29 by the square of 0·058.

5. Multiply the cube of 3·25 by the square root of $7\frac{1}{9}$.

Find the square root of

6. 18671041. 7. 277729. 8. 167281. 9. 1164241.

10. 66049. 11. 331776. 12. 112·1025.

13. Find the numerical value of $\sqrt{27} \times 3^3 \div 9^2$.

14. Find the square root of $54{\cdot}76 \times 0{\cdot}4356$.

15. What is the length of the side of a square field which contains $5\frac{5}{8}$ acres?

16. The area of a square is 31·36 square feet. What is the length of its side?

CHAPTER IV.

Mass and Weight. Ratio. Specific Gravity. Proportion. Variation. Percentages. Interest.

Mass and Weight.—The mass of a body is the amount of matter it contains; its weight is the force by which it is attracted by and to the earth. The mass is proportional to, and may be obtained from, the weight of a body.

The estimation of mass and weight may be made by means of some form of balance. The body, whose weight is required, is placed in one pan of a balance and known *weights* are added in the other; the sum of the weights used to counterpoise the body is the weight of the body.

The unit of mass or weight in the British system is the **Imperial Standard Pound**, and in the metric system the **kilogram**, which is equal to the weight of a cubic decimetre of water at a temperature of 4° C. [More accurately 1 kilog. = weight of 1·00003 cubic decimetres of water.]

Smaller units of weight are obtained by dividing the British unit into 12 or 16 equal parts, and larger units by multiplying it by 14, 112, 2240, as in the following tables:

Avoirdupois Weight.

437·5 grains or 16 drams = 1 ounce (oz.).
7000 grains or 16 ounces = 1 **pound** (lb.).
14 lb. = 1 stone (st.).
28 lb. = 1 quarter (qr.).
112 lb. = 1 hundredweight (cwt.).
2240 lb. = 20 cwt. = 1 ton.

Troy Weight. (Used by Jewellers.)

24 grains = 1 pennyweight (dwt.).
20 dwt. = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The connection between the two systems is that a grain avoirdupois and a grain troy are the same, but the number of grains in one pound is *not* the same, thus:

1 lb. avoirdupois = 7000 gr.
1 lb. troy = 5760 gr.

Metric Weight.

1 kilogram (kilog.) = 1000 grams (gm.).
1 gm. = 10 decigrams (dgm.) = 100 centigrams (cgm.).
= 1000 milligrams (mgm.).

1 gram is the weight of a cubic centimetre of water at a temperature of 4° C.

The British weights referred to are usually supplied in *sets*.

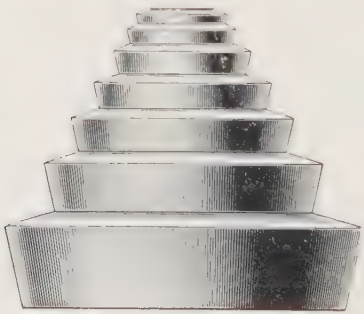


Fig. 47.—A set of ounce brass weights includes $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8 and 16 oz.

A set of ounce brass weights consists of the following $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8 and 16 oz.

Smaller weights consist of grains and fractions of a grain.

Larger weights, made of cast iron, are 1, 2, 3, 4, 7, 14, 28 and 56 lb. respectively.

Similarly, a set of brass **gram** weights includes 1, 2, 3, 5, 10, 20, 30, 50, 100, 200, 500 and 1000 grams.

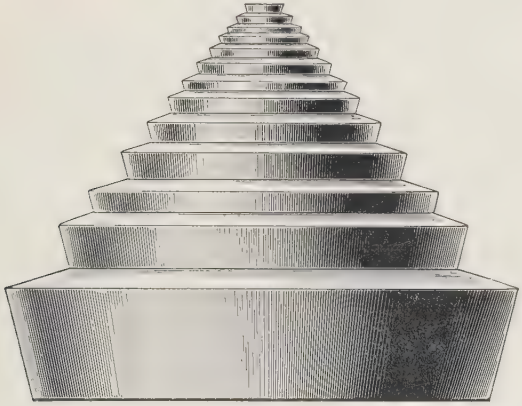


Fig. 48.—A set of gram weights includes 1, 2, 3, 5, 10, 20, 30, 50, 100, 200, 500 and 1000 grams.

Metric weights made of cast iron are 50, 100, 200 grams and the kilogram, which equals 1000 grams.

As 1 c.c. of water at 4° C. weighs 1 gram, there is, in the metric system, a simple relation between the unit of volume and the unit of weight.

A **litre** or 1000 c.c. of water weighs 1000 gr. or 1 kilog.

Conversion Table.

1 grain	= 0·0648 gram.
1 oz. avoirdupois	= 28·35 grams.
7000 grains = 1 lb.	= 453·6 grams.
1 ton	= $1{\cdot}016 \times 10^6$ grams.
1 gram	= 15·43 grains = 0·035 oz.
1 kilogram	= 2·2 lb.
1 millier or tonne	= 1·016 tons.

EXERCISES XX.

1. Find the number of pounds in 2·3 kilograms.

2. Find the number of pounds in 9·8 kilograms.

3. How many kilograms are there in 3·4 lb.?

4. Express (i) 9·7 kilograms in lb., (ii) 9·7 lb. in kilograms.

5. Find the number of tonnes in (i) 7·6 tons, (ii) 5·7 tons.

6. Express in tons (i) 8·4 tonnes, (ii) 9·4 tonnes.

7. A flask contains 2·5 litres of water, what is the weight of the water in grams?

8. A vessel contains 8·9 litres of water, what is the weight of the water in pounds?

9. Express a pressure of 19·3 kilog. per sq. cm. in lb. per sq. in.

10. Express a pressure of 13·8 lb. per sq. in. in kilog. per sq. cm.

11. The pressure on a piston is 13·5 kilog. per sq. cm. What is the pressure in lb. per sq. in.?

12. If 100 cubic inches of air weigh 31 grains, find in grams the weight of a cubic metre.

13. Find the number of grams in 120 lb.

14. Find (i) the number of grains in 100 grams, (ii) the number of pounds in 100 kilograms.

15. What is the number of pounds and kilograms in 0·7086 of a ton?

16. Express a pressure of 1 ton per square foot in kilograms per square metre.

Ratio.—When two quantities of the same kind are compared, the relation which one quantity bears to the other is called a **ratio.** If one quantity is 4 units and the other 16, then the ratio between the quantities may be expressed either as 4 : 16 or $\frac{4}{16}$. Or, the **ratio of the two quantities is the fraction that the first quantity is of the second.**

The numerator and denominator of the fraction are called the **terms** of the ratio, and as the value of a fraction is unaltered when both numerator and denominator are multiplied or divided by the same number (p. 24), it follows that a given ratio is unaltered in value when both terms are

multiplied or divided by the same number. Thus, multiplying or dividing $\frac{4}{16}$ by 4 we obtain $\frac{16}{64}$ and $\frac{1}{16}$, which respectively represent the same ratio as $\frac{4}{16}$.

EXAMPLE 1. *Find the ratio of 8 inches to 2 feet.*

As $2 \text{ ft.} = 24 \text{ in.};$

$$\therefore \text{ ratio} = \frac{8}{24} = \frac{1}{3}.$$

EXAMPLE 2. *Divide a line 5 inches long in the ratio of 2 : 3.*

Let AB (Fig. 49) denote a line 5 inches long. Measure off $AD = 2$ in. and $DB = 3$ in.; then the two lengths AD and DB are in the required ratio.

But

$AD = \frac{2}{5} AB$ and $DB = \frac{3}{5} AB$.

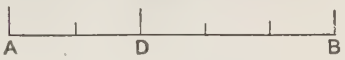


FIG. 49.

Hence, to divide a given number in a required ratio, add the two terms of the ratio to obtain a common denominator, and take each number in turn as numerators.

EXAMPLE 3. *Divide a line c units long in the ratio a : b.*

The ratio $a : b$ is measured by the fraction $\frac{a}{b}$.

Let AB (Fig. 50) denote a length of c units, measure off $AD = a$ and $DB = b$; then $AD = \frac{a}{c}$ and $DB = \frac{b}{c}$; but $c = a + b$;

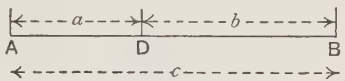


FIG. 50.

$$\therefore AD = \frac{a}{a+b}, \quad DB = \frac{b}{a+b}.$$

EXAMPLE 4. *Divide £70 in the ratio 2 : 3.*

Here the denominator becomes $2 + 3 = 5$.

$\frac{2}{5}$ of $70 = 28$; and $\frac{3}{5}$ of $70 = 42$.

The required amounts are £28 and £42.

When required to divide a given number in the ratio of two or more fractions, it is necessary to express the given fractions with a common denominator. Then, as fractions with the same denominators are in the same ratios as their numerators, it is only necessary to consider the numerators.

EXAMPLE 5. *Divide £78 in the ratios of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.*

The required amounts are obviously not $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of 78, and it is necessary to express the given fractions as fractions with a common denominator and to write $\frac{1}{2}$ as $\frac{6}{12}$, $\frac{1}{3}$ as $\frac{4}{12}$ and $\frac{1}{4}$ as $\frac{3}{12}$.

Then it is only necessary to divide 78 in the ratios of the numerators 6, 4 and 3, and the required amounts are

$$\frac{6}{13} \text{ of } 78 = £36, \quad \frac{4}{13} \text{ of } 78 = £24 \text{ and } \frac{3}{13} \times 78 = £18.$$

EXAMPLE 6. *Divide 10 guineas between A and B so that A receives 1 guinea for each half-crown that B gets.*

Here A receives $21s. = 42$ sixpences.

While B receives $2\frac{1}{2}s. = 5$ sixpences.

Hence the shares are as 42 : 5.

$$\therefore \ A\text{'s share} = \frac{42}{47} \text{ of } 10{\cdot}5 = £9.\ 7s.\ 8d.$$

$$B\text{'s share} = \frac{5}{47} \times 10{\cdot}5 = £1.\ 2s.\ 4d.$$

EXAMPLE 7. *A bankrupt owes three creditors £240, £360 and £375 respectively. If £325 is divided among them in the ratios of their claims, how much will each creditor receive and how much will it amount to in the £?*

$$240 + 360 + 375 = 975.$$

Hence the amounts received are

$$\frac{240}{975} \times 325 = £80,$$

$$\frac{360}{975} \times 325 = £120,$$

$$\frac{375}{975} \times 325 = £125.$$

Also $$\frac{325 \times 20}{975} = 6\frac{2}{3}s. = 6s.\ 8d. \text{ in the £1.}$$

EXAMPLE 8. *A has £210 and B has £140 in a business; divide a profit of £85 between them.*

$$A\text{'s share} = \frac{210}{350} \times 85 = £51,$$

$$B\text{'s share} = \frac{140}{350} \times 85 = £34.$$

EXAMPLE 9. *A has £210 in a business for a year and B has £140 in it for 7 months, divide a profit of £35 between them.*

The shares will be in the ratios of 210×1 and $\frac{140 \times 7}{12}$ or 2520 to 980, or 126 to 49.

$$\therefore \ A\text{'s share} = \frac{126}{175} \times 35 = £25\frac{1}{5} = £25.\ 4s.$$

$$B\text{'s share} = \frac{49}{175} \times 35 = £9\frac{4}{5} = £9.\ 16s.$$

EXERCISES XXI.

1. What is meant by ratio? How would you express the ratio of 5 tons to 8 lb.?

2. If the charge for carrying 221 tons of coal 66 miles be £23, how far ought 121 tons 11 cwt. to be carried for the same price?

3. A property worth £5880 is left to three persons whose shares are to be in the proportion of 3 : 5 : 7. How much will each receive?

4. Divide £28900 among 4 persons in parts proportional to 8, 5, 3, 1.

5. Divide £10200 among 4 persons in the ratio 7 : 5 : 3 : 2.

6. Divide £8026. 7*s.* 1*d.* between 4 persons in the ratio 3 : 4 : 7 : 11.

7. Cement and sand in the ratio 7 : 3 make up a weight of 80 lb. If 10 lb. of sand is added, what will then be the ratio of cement and sand?

8. An alloy is composed of three metals in the proportion of 10, 3 and 1; what weight of each is there in 150 lb. of alloy?

9. If a penny weighs $\frac{1}{3}$ oz. avoirdupois and is an alloy consisting of 95 per cent. copper, 4 per cent. tin and 1 per cent. zinc, what weight of each metal is necessary to form 240 pennies?

10. Two men carry a load of 3 cwt. 14 lb. between them, and the ratio of their loads are as 4 : 3. By how much does one load exceed the other?

11. One kind of gunpowder is made of 15 parts nitre, 2 parts sulphur and 3 parts charcoal. Another kind is made of 77 parts nitre, 9 parts sulphur and 14 parts charcoal. If 112 lb. of each kind be mixed together, find the amount of each of the three constituents in the mixture.

12. A rectangular field is 154·5 metres long by 103·75 metres wide; find its cost at 2·73 francs per square metre.

If the cost of putting a fence round the field were 2·73 francs per metre, find the ratio of the cost of the fence to that of the field.

13. What weight of copper must be added to 560 lb. of tin to make an alloy consisting of $4\frac{1}{4}$ parts copper to 1 tin?

14. The time of oscillation of a pendulum is proportional to the square root of its length, and a pendulum which beats seconds is 39·14 in. long. How many beats will be made in a minute by a pendulum 26·1 in. long?

Specific Gravity.—The specific gravity of a substance (usually designated sp. gr. or by the letters S.G.) is the *ratio* of the weight of any given volume of a substance to the weight of an equal volume of water.

Thus, if the weight of a given volume of cast iron is found to be 7·2 times that of the same volume of water, then the specific gravity of cast iron is 7·2. As the weight of a cubic inch of water is 0·036 lb., the weight of a cubic inch of cast iron is $0{\cdot}036 \times 7{\cdot}2 = 0{\cdot}26$ lb.

$$\text{Specific gravity} = \frac{\text{weight of any volume of a substance}}{\text{weight of an equal volume of water}}.$$

The specific gravity of mercury is 13·6, hence mercury is 13·6 times as heavy as water, and the weight of 1 c.c. of mercury is 13·6 gm.

Similarly, if the specific gravity of copper is 8·9, then 1 c.c. weighs 8·9 gm.

If V is the volume of a substance in cubic inches and S its specific gravity, then its weight is

$$V \times S \times 0{\cdot}036 \text{ lb.},$$

where 0·036 lb. is the weight of one cubic inch of water.

If v is the volume in cubic centimetres, and S the specific gravity, the weight is

$$v \times S \text{ grams.}$$

The specific gravity of a body may be obtained by weighing the body in air and afterwards in water. The difference in the weights is the weight of an equal volume of water, hence the specific gravity is given by

$$\text{Specific gravity} = \frac{\text{weight of body in air}}{\text{weight in air} - \text{weight in water}}.$$

Example 1. *A piece of metal weighs 50·5 lb. in air and 43·5 lb. in water. Find its specific gravity.*

$$\text{Specific gravity} = \frac{50{\cdot}5}{50{\cdot}5 - 43{\cdot}5} = \frac{50{\cdot}5}{7} = 7{\cdot}2.$$

Example 2. *A piece of ivory, specific gravity 1·875, weighs 6 lb. in air. Find its weight in water.*

Let w denote its weight in water, then

$$1{\cdot}875 = \frac{6}{6 - w};$$

$$\therefore\ 1{\cdot}875 \times 6 - 1{\cdot}875w = 6,$$

$$\text{or}\quad 1{\cdot}875w = 1{\cdot}875 \times 6 - 6 = 0{\cdot}875 \times 6;$$

$$\therefore\ w = \frac{0{\cdot}875 \times 6}{1{\cdot}875} = 2{\cdot}8 \text{ lb.}$$

EXAMPLE 3. *If a cubic foot of ice weighs 56 lb. and a cubic foot of water 62·3 lb., to what depth will a cubic foot of ice sink in water?*

The upward pressure of the water equals the weight of water displaced. Hence the block will sink a distance $\frac{56}{62\cdot3}$ of its height,

$$\text{or} \quad \frac{56\times1}{62\cdot3}=0\cdot8989 \text{ ft.}$$

It should be noticed that the number 0·8989 denotes the specific gravity of the ice.

TABLE I.

Specific Gravity.

Name.	Weight of 1 cubic inch in pounds.	Specific gravity.	Name.	Weight of 1 cubic inch in pounds.	Specific gravity.
Water	0·036	1	Antimony	0·24	6·7
Wrought iron	0·28	7·8	Aluminium	0·09	2·6
Cast iron	0·26	7·2	Oak	0·03	0·93
Steel	0·29	7·9	Mahogany	0·031	0·85
Brass	0·29	8·1	Pitch pine	0·025	0·7
Copper	0·32	8·8	Fir	0·019	0·53
Lead	0·41	11·4	Teak	0·035	0·98
Tin	0·27	7·4	Mercury	0·49	13·6
Zinc	0·26	7·2			

EXERCISES XXII.

1. A piece of metal weighs 120 grams in air and 100 grams in water. Find its sp. gr.

2. A piece of metal, sp. gr. 4, is found to weigh 1200 grams in air. What will be its apparent weight in water?

3. A bottle full of water is counterpoised by 983 grams in addition to the counterpoise of the empty bottle, and by 773 grams when full of alcohol. What is the sp. gr. of alcohol?

4. What is the sp. gr. of a certain substance, 20 cubic inches of which weighs 3 lb.?

5. A glass ball which weighs 1000 grams, has a weight of 630 grams in water and 650 grams in wine. Find the sp. gr. of wine.

6. When the mercurial barometer stands at 30·61 inches, the glycerin barometer stands at 329·2 inches. Given the sp. gr. of mercury 13·6, find the sp. gr. of glycerin.

7. A block of metal, sp. gr. 7·5, when immersed in water weighs 390 lb. What is its weight in air?

8. Three pints of a liquid, sp. gr. 0·6, are mixed with four pints of a liquid, sp. gr. 0·8, and there is no contraction. Find the sp. gr. of the mixture.

9. A piece of cork, sp. gr. 0·25, weighs 1 lb. What is the weight of a piece of brass, sp. gr. 8, which when tied to the cork will just sink it?

10. A piece of brass weighs 57 grams in air, 50 grams in water and 48·5 grams in glycerin. Find the sp. gr. of brass and of glycerin.

11. A piece of metal weighs 5 oz. in air and 4·3 oz. in water. Find its sp. gr.

12. A piece of gold weighs 5 oz. in air: (i) 4·74 oz. in water; (ii) 4·52 oz. in sulphuric acid; (iii) 4·68 oz. in hydrochloric acid; (iv) 4·79 oz. in alcohol. What is the sp. gr. of each liquid?

Proportion. When two ratios are equal, the four terms are said to be **proportional.** The equality of the ratios 2 : 3 and 12 : 18 may be expressed either as $2 : 3 :: 12 : 18$ or as $2 : 3 = 12 : 18$. This is read as 2 is to 3 as 12 is to 18, or 2 to 3 equals 12 to 18.

As the first two terms form a ratio, they must be quantities of the same kind. Similarly, the remaining two terms must be like quantities.

The outside terms 2 and 18 are called the **extremes,** the inner terms the **means.**

As $2 \times 18 = 3 \times 12$ the relation between the terms is such that **the product of the means is equal to the product of the extremes.**

Let a, b, c, d be the four terms of a proportion, then

$$a : b = c : d, \quad \text{or} \quad \frac{a}{b} = \frac{c}{d}.$$

Multiply each ratio by bd, then

$$\frac{abd}{b} = \frac{cbd}{d}, \quad \text{or} \quad ad = cb.$$

From this relation any single term can be found when the remaining three are given.

EXAMPLE 1. *If the first three terms of a proportion are* 6, 8 *and* 15, *what is the remaining term?*

Let F denote the fourth term ;

$$\therefore\ 6 : 8 = 15 : F, \text{ or } F = \frac{8 \times 15}{6} = 20.$$

Third Proportional.—When the two means of a proportion are equal, the fourth term is called a **third proportional.**

EXAMPLE 2. *Find a third proportional to* 6, 8 *and* 8.

If T denote the third proportional, then we obtain the relation

$$6 : 8 = 8 : T,$$

i.e. we have to find a number which has the same ratio to 8 that 8 has to 6.

$$\therefore \frac{6}{8} = \frac{8}{T},$$

$$\text{or } T = \frac{64}{6} = 10\tfrac{2}{3}.$$

Hence, to find a third proportional, multiply the second number by itself and divide by the first.

Mean Proportional.—The mean proportional to two numbers is found by taking the square root of their product.

EXAMPLE 3. *Find a mean proportional to* 1 *and* 16.

$$\text{Mean proportional} = \sqrt{1 \times 16} = \sqrt{16} = 4.$$

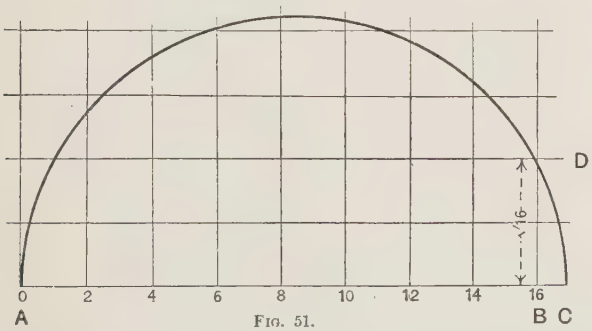


FIG. 51.

Graphically. Using squared paper set off 16 units from A to B (Fig. 51) and 1 unit from B to C. On AC describe a semicircle. Draw BD perpendicular to AC and intersecting the semicircle in D. Then BD is the mean proportional required.

Inverse Proportion. One quantity is said to be inversely proportional, or to *vary inversely*, as another, when an increase in the value of one produces a decrease in that of the other.

EXAMPLE. *Plot the curve* $y=\frac{1}{x}$ *from* $x=1$ *to* $x=8$; *find from the curve the value of* y (i) *when* $x=2{\cdot}5$, (ii) *when* $x=6{\cdot}5$.

When $x=2$, $y=\frac{1}{2}=0{\cdot}5$; when $x=4$, $y=0{\cdot}25$; other values of x may be assumed and corresponding values of y obtained and tabulated.

x	1	2	3	4	5	6	7	8
y	1	0·5	0·33	0·25	0·2	0·167	0·143	0·125

Plot these values as in Fig. 52 and draw a fair curve through the plotted points. Then the value of y for any intermediate point can be obtained. Thus (i) when $x=2{\cdot}5$, $y=0{\cdot}4$, (ii) when $x=6{\cdot}5$, $y=1{\cdot}54$; the former is indicated by the point F, the latter by the point G.

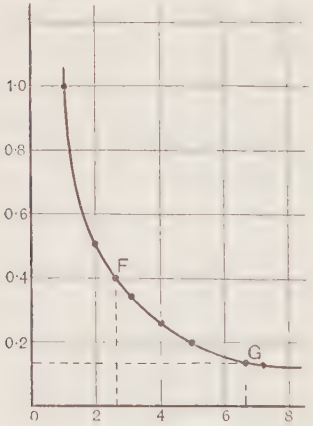


FIG. 52.—Graph of $y=\frac{1}{x}$.

Variation.—What is called variation deals with all cases in which the magnitude of one quantity depends on that of another, and is in reality the theory of proportion in a much more general form.

The symbol used to denote variation is $\propto$; thus $a \propto b$ is read as a varies as b.

For example, if the diameter of a circle be doubled or halved then the circumference of the circle is doubled or halved. This fact is expressed by $c \propto d$, where c is the circumference and d the diameter of a circle.

EXAMPLE 1. *The circumference of a circle varies as its diameter. When the diameter is* 17 *in., the circumference is* 53·4 *in. Find the circumference of a circle* 29 *in. diameter.*

As $c \propto d$; $\therefore\ c=kd$, ...(i)

where k is a constant the value of which may be found from given values of c and d.

Substituting; $\therefore\ 53{\cdot}4 = k \times 17,$

$$\text{or}\quad k = \frac{53{\cdot}4}{17} = 3{\cdot}142.$$

Hence, the relation between the circumference of a circle and its diameter is $c = 3{\cdot}142 \times d$.

Thus, when $d = 29$, $c = 3{\cdot}142 \times 29 = 91{\cdot}1$ in.

Example 2. *The area of a circle varies as the square of its diameter. The area is* 95·1 *sq. in. when the diameter is* 11 *in. What is the area of a circle* 58 *in. diameter?*

If A denotes the area and d the diameter, then $A \propto d^2$;

$$\therefore\ A = kd^2.$$

Substitute the given values;

$$\therefore\ 95{\cdot}1 = k \times 11^2,$$

$$\text{or}\quad k = \frac{95{\cdot}1}{121}.$$

Hence, the required relation is $A = \frac{95{\cdot}1}{121} \times d^2$.

When d is 58, $\quad A = \frac{95{\cdot}1}{121} \times 58^2 = 2643{\cdot}9$ sq. in.

Example 3. *The volume of a sphere varies as the cube of its diameter. If the volume of a sphere* 6 *in. diameter is* 113·2 *cub. in., what is the volume of a sphere* 3·5 *in. diameter?*

Let v denote the volume and d the diameter of the sphere, then

$$v \propto d^3,$$

$$\text{or}\quad v = kd^3.$$

Substituting the given values,

$$113{\cdot}2 = k \times 6^3;$$

$$\therefore\ k = \frac{113{\cdot}2}{216}.$$

Hence, the relation is $\quad v = \frac{113{\cdot}2}{216} \times d^3,$

and for any given value of v or d the remaining value can be found; thus, when $d = 3{\cdot}5$,

$$v = \frac{113{\cdot}2}{216} \times (3{\cdot}5)^3 = 22{\cdot}48 \text{ cub. in.}$$

EXAMPLE 4. *Given that y is proportional to x, and that when y is 7, x is 8, find the value of y when x is 5·6.*

The statement that y is proportional to x may be written $y = kx$.

The value of the constant k can be obtained as in the example by substituting the given values of x and y;

$$\therefore\ 7 = k \times 8, \quad \text{or} \quad k = \tfrac{7}{8}.$$

Thus, the required relation between x and y is given by

$$y = \tfrac{7}{8}x.$$

When $x = 5{\cdot}6$, $\qquad y = \tfrac{7}{8} \times 5{\cdot}6 = 4{\cdot}9.$

The relation between x and y is best shown by using squared paper; thus, when $x = 8$, $y = 7$; and when $x = 0$, $y = 0$.

Hence, set off on squared paper the point A ($x = 8$, $y = 7$) and join A to 0 by a straight line (Fig. 53); then for any given value of x less than

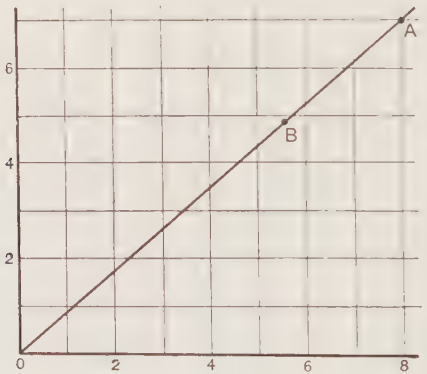


FIG. 53.

8, the value of y can be found. Thus, when x is 5·6, the point B is obtained where $y = 4{\cdot}9$. Similarly, when $x = 4$, $y = 3{\cdot}5$.

Simple and Compound Proportion.—Questions dealing with so-called simple and compound proportion are readily solved by using letters to denote the various terms and finally substituting to obtain numerical values.

EXAMPLE 1. *If* 15 *men can dig* 150 *yards of trench in* 6 *days, how many men will be required to dig* 70 *yards in the same time?*

Let m denote the number of men and y the number of yards; then m varies as y,

$$\text{or} \quad m = ky.$$

Substituting the values $m=15$ and $y=150$, then

$$15 = k \times 150 \quad \text{or} \quad k = \tfrac{1}{10}.$$

Hence, the required relation is $m = \frac{y}{10}$.

When $y=70$, $m=7$. Make the point F (Fig. 54) to denote 15 and 150, and join F to the origin O. Then the number of men required to dig 70 yards is seen to be 7. Also the number of men for any number of yards up to 150 can be read off from the graph.

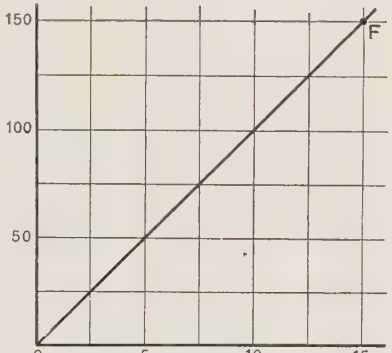


FIG. 54.

EXAMPLE 2. *If* 10 *men working* 9 *hours per day can dig a trench* 30 *yards long in two days, how many men working* 8 *hours per day will be required to dig a trench* 120 *yards long in* $4\frac{1}{2}$ *days?*

Let m denote the number of men, h the number of hours, d the number of days and y the number of yards.

Then the number of men will be directly proportional to the number of yards in the trench and inversely as the number of days and the number of hours per day;

$$\therefore \quad m = k\frac{y}{dh}.$$

Substituting, $$10 = k \times \frac{30}{2 \times 9}; \quad \therefore \quad k = 6.$$

Hence, the relation is $$m = \frac{6 \times y}{dh}.$$

Thus, when $y=120$, $d=4\frac{1}{2}$ and $h=8$;

$$\therefore \quad m = 6 \times \frac{120}{4\frac{1}{2} \times 8} = 20.$$

EXAMPLE 3. *The total yearly expense of a school for 175 boys is £3675. What will be the expense when the number of boys is 100?*

If E denotes the expense and n the number of boys, then

$$E = kn.$$

Substituting, $3675 = k \times 175$;

$$\therefore\ k = 21.$$

Hence, the relation is $E = 21n$.

When $n = 100$, $E = 21 \times 100 = £2100$. Set out on squared paper the point F (Fig. 55), to denote 3675 and 175, and join the point F to the origin O. Then the expense for 100 boys is seen at A to be £2100.

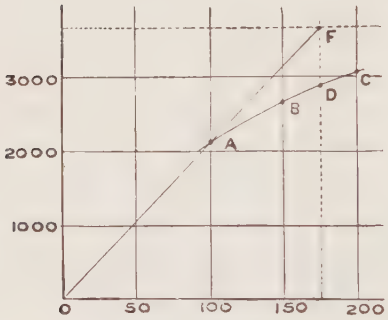


FIG. 55.

Although it is frequently convenient to assume that various values are proportional, such is not necessarily the case. Thus, if the cost of sending a ton of coals from one place to another is 7*s.*, the cost of sending 10 tons the same distance is probably less than 70*s.* and still less per ton for a larger quantity.

In the Parcel Post the cost of transmission for 1 lb. is 3*d.*, but for 5 lb. the cost is 7*d.* and not $5 \times 3d.$ Again, if the cost of a school for 100 boys is known, the amount will not diminish to one half when the number is 50, nor become twice the amount when the number of boys is 200. There are certain expenses, such as rent and taxes, which remain the same whether the number of boys is increased or diminished. The numbers denoting the cost and the number of boys may be plotted on squared paper, and do not lie on a straight line, but on a curve.

EXAMPLE 4. *The total yearly expenses in keeping a school are known to be as follows:*

£2100 for 100 boys.

£2650 for 150 boys.

£3050 for 200 boys.

What is the probable expense for 175 boys?

The three points A, B and C are set off on squared paper to denote the three given cases (Fig. 55).

A straight line cannot be made to pass through the three given points. But a curve may be made to pass through the three points by using a flexible strip of wood, or metal. Then, corresponding to 175, we obtain at D the total yearly expense to be £2860.

EXERCISES XXIII.

1. If 60 men can build a wall in 8 days working 10 hours a day, how many men can build a wall of the same size in 20 days working $7\frac{1}{2}$ hours a day?

2. If 10 men working 9 hours a day can dig a trench 30 yards long in 2 days, in how many days will 40 men dig 120 yards working 8 hours a day?

3. How far at the same rate per mile can 96 tons be carried for the price paid for carrying 72 tons a distance of 288 miles?

4. If 90 men can dig a ditch 50 yd. long, 6 ft. wide and 12 ft. deep in $4\frac{1}{2}$ days, how many men can dig a ditch 240 ft. long, 2 yd. wide and 4 ft. deep in 18 days?

5. If 18 men working 10 hours a day can build a wall 8 ft. high and 120 ft. long in 3 days, how long will it take 15 men working 7 hours a day to build a wall 9 ft. high and 84 ft. long?

6. If in the last question the bricks are laid in mortar and the space per brick is 9 in. long, 3 in. thick and $4\frac{1}{2}$ in. wide, how many bricks will be required for each wall if the thickness of the wall is 9 in.? What will be the cost at £1. 5*s.* per thousand?

7. If 13 men in 5 weeks earn £97. 10*s.*, in what time will 18 men working at the same rate earn £297?

Percentages.—In the addition, subtraction or comparison of two or more vulgar fractions, it is necessary to express the fractions in the forms of equivalent fractions

having a common denominator. When such a denominator is 100, the fractions are called **percentages**; hence, **when one term of a ratio is 100, the other term expresses the rate per 100, or the rate per cent.**

Any given fraction may be expressed in this form: thus, $\frac{1}{4} = \frac{25}{100}$, or 25 per cent.; $\frac{1}{2} = \frac{50}{100}$, or 50 per cent.: and $\frac{3}{4} = \frac{75}{100}$, or 75 per cent.

Instead of writing the words *per cent.*, the symbol % is frequently used and has the same meaning. Thus, the preceding cases could be written as 25 %, 50 % and 75 % respectively.

As 5 % or $\frac{5}{100} = \frac{1}{20}$, it follows that 5 % of £1 = 1*s.* and 2·5 per cent. of £1 is 6*d.*

EXAMPLE 1. *Express* 3*s.* 4*d.* *as a percentage of* £3. 6*s.* 8*d.*

$$3s.\ 4d. = 3\tfrac{1}{3}s.$$

Also $$£3.\ 6s.\ 8d. = 66\tfrac{2}{3}s.;$$

$$\therefore \text{ percentage} = \frac{3\frac{1}{3}}{66\frac{2}{3}} \times 100 = 5.$$

EXAMPLE 2. $12\frac{1}{2}$ *per cent. of a certain sum is* 2*s.* 6*d.* *Find the sum.*

$$2s.\ 6d. = 30d.;$$

$$\therefore\ 1\% = \frac{30}{12{\cdot}5}.$$

$$\text{Sum required} = \frac{30 \times 100}{12{\cdot}5} = 240d. = £1.$$

EXAMPLE 3. *The percentage composition of a certain alloy is* 60 *per cent. copper,* 25 *per cent. zinc,* 10 *per cent. tin and* 5 *per cent. lead. Find the weight of each metal required to make* 200 *lb. of the alloy.*

Amount of	copper	$= 200 \times \frac{60}{100}$	= 120 lb.
,,	zinc	$= 200 \times \frac{25}{100}$	= 50 ,,
,,	tin	$= 200 \times \frac{10}{100}$	= 20 ,,
,,	lead	$= 200 \times \frac{5}{100}$	= 10 ,,
		Total,	200 lb.

Adding the various amounts together, as shown, serves as a check on the calculated values.

EXAMPLE 4. *A man sells* 10 *cwt. of sugar at* $2\frac{3}{4}d.$ *per pound, thereby gaining* 11*s.* 8*d. What is his profit per cent.?*

$$\begin{aligned} 1120 \text{ lb. at } 2\tfrac{3}{4}d. &= £12.\ 16s.\ 8d. \\ \text{Profit} &= \underline{\quad\quad 11s.\ 8d.} \\ \text{Cost} &= £12.\ 5s.\ 0d. \end{aligned}$$

$$\text{Profit} = \frac{11s.\ 8d.}{£12.\ 5s.} \times 100 = 4{\cdot}8\ \%.$$

EXAMPLE 5. *A bought a house for* £350 *and sold it to B for* £400; *B afterwards sold it to C for* £350. *How much did A and B gain or lose by the transaction?*

$$A \text{ gained } \frac{50}{350} \times 100 = 14\tfrac{2}{7}\ \%.$$

$$B \text{ lost } \frac{50}{400} \times 100 = 12\tfrac{1}{2}\ \%.$$

EXAMPLE 6. *The population of a certain town in* 1891 *was* 25000, *and in* 1901, 40000. *The population of another town in* 1891 *was* 55000, *and in* 1901, 70000. *Express as a percentage the rate of increase of each of the two towns.*

In both towns there is an increase of 15000, but the two towns are not increasing at the same rate.

Thus the first town gains 15000 on a population of 25000, hence the increase is $\frac{15000}{25000} \times 100 = 60\ \%$.

In the second town the percentage increase is given by

$$\frac{15000}{55000} \times 100 = 27{\cdot}3\ \%.$$

A percentage also enables a comparison to be made between two or more given results; thus, if two experimental values are determined and found to be 0·00013 and 0·00015, the difference between them, 0·00002, fails to give an idea as to the accuracy obtained. Thus, if the latter result is known to be fairly accurate, the former is in error by 2 in 13, or 9 per cent nearly.

Percentage of a Percentage.—In many cases a percentage of a given percentage is required, and is obtained by the usual process of the multiplication of fractions.

EXAMPLE 7. *In a field of* 5 *acres it is found that* 35 *per cent. is meadow land and the remainder arable land. If* 15 *per cent. of the latter is waste, what is the area of the waste?*

As 35 per cent. is meadow the remainder 65 % is arable;

$$\therefore \text{ area of waste} = \frac{65}{100} \times \frac{15}{100} \times 5 = 0{\cdot}4875 \text{ acre.}$$

EXERCISES XXIV.

1. If the increase in the length of a flat steel bar 8 in. long is 20^{-3}, what will be the length of the bar?

2. A certain kind of brass contains 65 per cent. copper and 35 per cent. zinc. Find the amount of each metal in 112 lb. of brass.

3. The composition of a certain kind of naval brass is 62 per cent copper, 37 per cent. zinc and 1 per cent. tin. Find the amount of each kind of metal required to form 400 lb. of the alloy.

4. If gunpowder is composed of 15 parts nitre, 2 sulphur and 3 charcoal, what is the percentage composition, and how many pounds of each are required in the manufacture of 112 lb. of gunpowder?

5. A tradesman buys 240 chairs for £160 and sells them at 16*s.* 8*d.* each. What is his profit per cent.?

6. A tradesman gains 30 per cent. by selling goods for 13*s.* How much would he gain per cent. by selling them for 10*s.* 9*d.*?

7. A merchant by selling goods for £120 gains 25 per cent. How much would he lose if he sells them for £88?

8. Divide £814 among three persons in the ratios $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$. Also state what percentage of the total amount each person receives.

9. One kind of gunpowder consists of 9 per cent. sulphur, 14 per cent. charcoal and 77 per cent. nitre. Another kind is made of 2 parts sulphur, 3 parts charcoal and 15 parts nitre. If 200 lb. of each be mixed together, find the amount of each of the three constituents in the mixture.

10. Divide £56 between *A*, *B*, *C* and *D* in the ratios of the numbers 3, 5, 7, 9. State what percentage of the total amount each person receives.

11. A sum of £2928 is to be divided among four persons in the ratios $\frac{2}{3} : \frac{3}{4} : \frac{4}{5} : \frac{5}{6}$. Find the share of each.
What percentage of the total amount does each person receive?

12. A log of timber 12 ft. 3 in. long, 10·75 in. wide and 10 in. thick weighs, when newly cut, 341 lb. If the wood loses 20 per cent. in weight while drying, determine the weight of a cubic foot when dry.

13. The yield of a gold-mine increases from 0·0043 per cent. to 0·0048 per cent.; how much more gold does a ton of ore now yield than before. [1 lb. = 7000 grains.]

14. If type-metal consists of 80 per cent. of lead and 20 per cent. of antimony, what will be the weight of a cubic foot of the alloy?

15. In a piece of coal there was found to be 11·3 lb. of carbon, 0·92 lb. of hydrogen, 0·84 lb. of oxygen, 0·56 lb. nitrogen, 0·71 lb. of ash. State the percentage composition of the coal.

16. If at a given instant the volume of a raindrop is 0·00035 cub. in. and if 0·25 per cent. of that volume is added per second, what will be its volume in 6 seconds?

17. How many people died in a town of 200,000 inhabitants in which the death rate was 17 per thousand?

18. A steel casting is to contain 0·2 per cent. of carbon. How much carbon will be required for 2 cwt. of metal?

19. In a certain kind of coal there is 4·9 per cent. of ash. What weight of ash will be produced when 20 tons of coal are burned?

Interest.—The payment made for the loan of a sum of money is called **interest.** Interest is calculated usually at the rate of so much per cent., the unit of time being one year.

Thus, if £100 is borrowed for one year at 5 per cent., then at the end of the year the payment to be made will be the original £100, called the **principal,** together with the interest which is $\frac{5}{100}$ of 100 or £5.

The sum of the interest and the principal lent for any specified period is called the **amount.** In the preceding instance the *amount* is £(100 + 5) = £105.

When the interest is paid at stated intervals, such as yearly, or half yearly periods, during the period of a loan, the money is said to be lent at **simple interest.** But when the interest is from time to time added to the principal, thus itself bearing interest and continually increasing the principal, then the sum is said to be lent at **compound interest.**

EXAMPLE 1. *Find the simple interest on* £500 *for* 7 *years at* 3 *per cent.*

Interest for 1 year $= 500 \times \frac{3}{100} =$ £15.
Interest for 7 years = £15 × 7 = £105.

The interest for a fraction of a year may be calculated by the usual rules for the multiplication of fractions.

EXAMPLE 2. *Find the simple interest on* £500 *for* (i) 7 *years* 3 *months,* (ii) 7 *years* 6 *months at* 3 %.

Interest for 1 year = £15.
7 years 3 months = $7\frac{1}{4}$ years.

(i) ∴ Interest for $7\frac{1}{4}$ yrs. $= 15 \times 7\frac{1}{4}$
$= 15 \times 7 + \frac{1}{4} \times 15 =$ £108. 15*s.*

(ii) Interest for $7\frac{1}{2}$ yrs. $= 15 \times 7\frac{1}{2} = 15 \times 7 + 15 \times \frac{1}{2}$
$=$ £112. 10*s.*

The interest for a fraction of a year may be obtained by inserting the number of days as the numerator of a fraction, the denominator being 365, the number of days in a year.

EXAMPLE 3. *Find the simple interest on* £800. 15*s.* *for* 3 *years* 10 *weeks and* 1 *day at* 5 *per cent.*

Interest for 1 year $= 800{\cdot}75 \times \frac{5}{100} =$ £40. 0*s.* 9*d.*

10 weeks and 1 day = 71 days ;

$\therefore$ 3 years and 71 days $= 3\frac{71}{365}$ years.

Interest $=$ £40. 0*s.* 9*d.* $\times 3\frac{71}{365} =$ £127. 16*s.*

EXERCISES XXV.

Find the simple interest on

1. £185 for 4 years at 4 per cent.
2. £345. 19*s.* 4*d.* for 4 years at 2 per cent.
3. £748. 15*s.* for $9\frac{1}{2}$ years at $5\frac{1}{2}$ per cent.
4. £926. 15*s.* for 7 years at 1 per cent.
5. €150 from September 20 to November 22 (73 days) at $4\frac{3}{4}$ per cent.

Find the amount of

6. £802. 17*s.* 6*d.* in 4 years at $2\frac{1}{2}$ per cent.
7. £12. 0*s.* $2\frac{1}{2}d.$ in 3 years 1 month at 1 per cent.
8. £2833. 6*s.* 8*d.* in 3 years at $4\frac{1}{2}$ per cent.
9. £350 in $4\frac{1}{2}$ years at $6\frac{5}{8}$ per cent.
10. Find to the nearest penny the simple interest on £475. 10*s.* for 69 days at $2\frac{1}{2}$ per cent. per annum.

CHAPTER V.

Indices. Logarithms. Multiplication and Division by Logarithms. Involution and Evolution by Logarithms. Calculation of Logarithms. Simple and Compound Interest. Discount. Present Worth. Stocks and Shares.

Indices.—As already explained (p. 8) the product 3×3 is written as 3^2, and $3 \times 3 \times 3$ is denoted by 3^3; also a^2 denotes a to the power 2, or a squared. Similarly, a^3 indicates a to the power 3. The small number at the right and near the top of a figure, or letter, denotes the number of times the figure, or letter, is repeated, and is called an index. When more than one index occur in a given expression they are called **indices.**

As $3^2 = 3 \times 3$

and $3^3 = 3 \times 3 \times 3,$

$$3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5,$$

or $3^2 \times 3^3 = 3^{2+3} = 3^5.$

Similarly, $3^4 \times 3^3 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3) = 3^7.$

This process may be expressed in a more general form:

$$a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5,$$

and $a^m \times a^n = a^{m+n}.$

The product of two or more powers of the same number, or quantity, has an index equal to the sum of these indices. This rule is called the **First Index Law.**

$$3^4 \div 3^3 = \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3^{4-3} = 3.$$

$$3^5 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} = 3^{5-2} = 3^3.$$

$$3^3 \div 3^3 = \frac{3 \times 3 \times 3}{3 \times 3 \times 3} = 3^{3-3} = 3^0 = 1.$$

$$a^4 \div a^2 = \frac{a \times a \times a \times a}{a \times a} = a^2.$$

$$a^m \div a^n = a^{m-n}.$$

The quotient of two powers of the same number has an index equal to the difference of the indices of dividend and divisor. This rule is called the **Second Index Law.**

$$(3^2)^3 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3^6.$$

$$(3 \times 2)^2 = 3^2 \times 2^2.$$

$$(3^2 \times 2^3)^2 = (3 \times 3)^2 \times (2 \times 2 \times 2)^2 = 3^4 \times 2^6.$$

Similarly, $(a \times b)^3 = a^3 \times b^3.$

$$(a \times b)^n = a^n \times b^n.$$

To raise a product to a power, raise each factor to the same power. This rule is called the **Third Index Law.**

Fractional Index. The root of a number may be expressed either by using the root sign, or by means of a fractional index.

Thus, the square root of 9 may be written either as $\sqrt{9}$ or $9^{\frac{1}{2}}$, the second expression being read as 9 to the power $\frac{1}{2}$.

Also $$\sqrt{9} \times \sqrt{9} = 9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9;$$

and $$27^{\frac{1}{3}} \times 27^{\frac{1}{3}} \times 27^{\frac{1}{3}} = 27^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 27.$$

$$3^{\frac{1}{2}} \times 3^{\frac{1}{3}} = 3^{\frac{3}{6}+\frac{2}{6}} = 3^{\frac{5}{6}}.$$

The operations indicated by a fractional index may in some cases be presented in a simpler form.

EXAMPLE 1. *Simplify* $8^{\frac{2}{3}}$.

Since $8^2=64$, $8^{\frac{2}{3}}$ may be written $\sqrt[3]{64}$.

Also $\sqrt[3]{64}=4\,;\quad\therefore\ 8^{\frac{2}{3}}=4.$

Similarly, $4^{1\cdot5}=4^{\frac{3}{2}}=\sqrt{64}=8.$

EXAMPLE 2. *Find the value of* $(16)^{2\cdot5}$.

$$16^{2\cdot5}=16^{\frac{5}{2}}=4^5.$$

Also $4^5=1024\,;\quad\therefore\ 16^{2\cdot5}=1024.$

These methods are only applicable to comparatively simple numbers. The numerical values of quantities in which indices of a more difficult character occur are readily obtained by logarithms.

Powers of 10.—Using the notation of indices, we may obtain some important results connected with the number 10.

Thus $10^2=100,\ 10^3=1000,\ 10^4=10000.$

Also $10^2\times10^3=10^{2+3}=10^5.$

$$10^{0\cdot5}\times10^2=10^{2\cdot5}.$$

$$10^4\div10^3=\frac{10\times10\times10\times10}{10\times10\times10}=10^{4-3}=10.$$

$$10^2\div10^2=\frac{10\times10}{10\times10}=10^{2-2}=10^0=1.$$

$$10^4\div10^4=10^{4-4}=10^0=1.$$

Any number may be written as a number lying between 1 and 10 multiplied by a power of 10. The power of 10 is obtained by *counting from the units figure, but not including it.*

$$29{,}000{,}000=2{\cdot}9\times10^7.$$

$$0{\cdot}00000029=2{\cdot}9\times10^{-7}.$$

$$5{,}280{,}000=5{\cdot}28\times10^6.$$

$$5280=5{\cdot}28\times10^3.$$

$$0{\cdot}00528=5{\cdot}28\times10^{-3}.$$

This concise method of representing numbers is very convenient in practice, more especially when dealing with comparatively large numbers and logarithms.

Logarithms.—A logarithm is an **index**; and as some of the most useful applications of indices are concerned with logarithms, it is necessary to consider carefully some of the terms used, and also their definitions.

The logarithm of a number to a given base is the index of the power to which that base must be raised to give the number.

[The abbreviation log is generally used.]

Any number may be used as a base; thus, if the base be 2, since $2^3 = 8$, then $\log_2 8 = 3$.

This result is read as log 8 to base 2 equals 3.

Similarly, if the base be 3, then, as $3^4 = 81$, $\log_3 81 = 4$, or, log 81 to base 3 equals 4.

The logarithms in general use are known as **common logarithms** or **logarithms to base 10.** Although other bases could be and are used, in the following pages and in practical work generally, the logarithms are assumed to be common logarithms unless the contrary is expressed.

Since 10 raised to the power 1 or $10^1 = 10$, 1 is the logarithm of 10 to the base 10.

Similarly, $10^2 = 100$, or 10 raised to the power 2 is 100;

$\therefore$ 2 is the logarithm of 100.

Again, $10^3 = 1000$, or 3 is the logarithm of 1000.

Since $10^0 = 1$, it follows that $\log 1 = 0$.

Although it is possible in such numbers as 100, 1000, etc., only to find the index exactly, it is always possible to find it accurately to a given number of decimal places. The fractional part of a logarithm is called the **mantissa**, and the integral part the **characteristic.** The characteristic depends for its numerical magnitude on the power of 10 in the given number. The mantissae consist of a series of decimal fractions, of 3, 4, 5, 6, 7 or more figures, arranged in Tables, and are called three-figure, four-figure, five-figure, etc., logarithms respectively.

Logarithms such as those in Table II. are known as four-figure logarithms. In the tables of common logarithms, it is only necessary to insert the mantissae; the characteristic for any given number may easily be inserted by inspection. This process is more fully explained on p. 102.

As all these logarithms are powers of 10, and the base is 10, our definition becomes: **the logarithm of a number denotes the power to which 10 must be raised to give the number.**

As the logarithm of 1 is 0, and $\log 10 = 1$, the logarithms of all numbers between 1 and 10 will consist wholly of decimals.

Thus, if 10 were raised to the power 0·301, the result would be 2;

$$\therefore \quad 10^{0\cdot301} = 2.$$

Also $\qquad 10^{0\cdot4771} = 3, \quad 10^{0\cdot6021} = 4, \quad 10^{0\cdot699} = 5,$ etc.

As logarithms are indices or powers of 10, the preceding rules of indices are applicable to logarithms.

Thus $\qquad 10^{0\cdot301} \times 10^{0\cdot6021} = 2 \times 4,$

or $\qquad 10^{0\cdot301+0\cdot6021} = 10^{0\cdot9031} = 8.$

Referring to Table II. it will be found that the mantissa of $\log 2 = 3010$, of 4 is 6021, and of 8 is 9031.

Hence we may obtain the following definitions:

The logarithm of a product is equal to the sum of the logarithms of its factors.

Again, as $\quad 10^{0\cdot6021} \div 10^{0\cdot301} = 10^{0\cdot6021-0\cdot301} = 10^{0\cdot301} = 2.$

The logarithm of a quotient is found by subtracting the logarithm of the divisor from the logarithm of the dividend.

The logarithm of a power is equal to the logarithm of the number multiplied by the index of the power.

The logarithm of a root is obtained by dividing the logarithm of the number by the index of the root.

It should be noticed that the last rule is really included in that preceding it. Thus, $\sqrt[3]{8}$, or $8^{\frac{1}{3}}$, simply denotes that 8 is to be raised to the power $\frac{1}{3}$.

Referring to Table II. it will be found that a series of numbers are printed on the extreme left of the page, *e.g.* the number 3010 is immediately opposite to and in the same horizontal line as the number 20.

Now $\qquad 20 = 2 \times 10^1;$

$$\therefore \quad \log 20 = 1{\cdot}3010.$$

Also $2 = 2 \times 10^0$;

$\therefore \log 2 = 0{\cdot}3010.$

Similarly, $200 = 2 \times 10^2$;

$\therefore \log 200 = 2{\cdot}3010.$

$2000 = 2 \times 10^3.$

$\therefore \log 2000 = 3{\cdot}3010.$

Also $\log 3 = 0{\cdot}4771$, $\log 30 = 1{\cdot}4771$,

$\log 300 = 2{\cdot}4771$, $\log 3000 = 3{\cdot}4771.$

When a number is greater than unity, the characteristic of its logarithm is one less than the number of digits in the number. Thus, in 300 there are three digits and the characteristic of its logarithm is 2.

This simple method of ascertaining the characteristic of the logarithm of a number may also be applied when the number is less than unity.

Thus $\log 40 = 1{\cdot}6021.$

$\log 4 = 0{\cdot}6021.$

$\log 0{\cdot}4 = \bar{1}{\cdot}6021.$

$\log 0{\cdot}04 = \bar{2}{\cdot}6021.$

$\log 0{\cdot}004 = \bar{3}{\cdot}6021.$

It will be noticed that as $0{\cdot}4 = 4 \times 10^{-1}$, $0{\cdot}04 = 4 \times 10^{-2}$, and $0{\cdot}004 = 4 \times 10^{-3}$, and so on, the characteristic in each case is negative, but as the mantissa of the logarithm is necessarily positive, the negative sign is placed over the top of the characteristic instead of in front of it.

The rule for finding the characteristic may be expressed as follows:

The characteristic of the logarithm of a number less than unity is negative and is one greater than the number of ciphers preceding the first significant figure.

How to find the Logarithm of a Number.—The numbers 1 to 9 are not given in Table II., but the mantissa of their logarithms is in each case the same as those of numbers ten times as great.

Any number which consists of two figures is found on the extreme left of the table and the mantissa of its logarithm in the next column and directly opposite to it. Thus,

$$\log 23 = 1{\cdot}3617, \quad \log 2{\cdot}3 = 0{\cdot}3617,$$
$$\log 81 = 1{\cdot}9085, \quad \log 8{\cdot}1 = 0{\cdot}9085.$$

In addition to the numbers 10 to 99 already referred to, there are at the top of the table, the numbers, 0, 1 ... 9; and on the extreme right of the page a table of differences is given; by means of these numbers and differences the logarithm of a number consisting of three or four figures can be obtained.

EXAMPLE 1. *Find* (i) *log* 23, (ii) *log* 236.

	0	1	2	3	4	5	6	7	8	9	Differences.		
											1 2 3	4 5 6	7 8 9
23	**3617**	3636	3655	3674	3692	3711	**3729**	3747	3766	3784	2 4 6	7 9 11	13 15 17

(i) First look at the extreme left of the table for the number 23, then in the column marked 0, and in the same horizontal row as 23, the four figures 3617 are found. This is the mantissa of the logarithm, and as 23 is $10^1 \times 2{\cdot}3$ the characteristic is 1.

$$\therefore \log 23 = 1{\cdot}3617.$$

(ii) In the horizontal row of figures corresponding to number 23 and the vertical column 6 we find the number 3729. This is the mantissa, and as $236 = 10^2 \times 2{\cdot}36$ the characteristic is 2. Hence, $\log 236 = 2{\cdot}3729$.

Similarly, $236000 = 2{\cdot}36 \times 10^5$;

$$\therefore \log 236000 = 5{\cdot}3729.$$

Also $0{\cdot}000236 = 2{\cdot}36 \times 10^{-4}$;

$$\therefore \log 0{\cdot}000236 = \bar{4}{\cdot}3729.$$

It will be seen that the mantissa remains the same for all numbers formed by the three figures 236.

EXAMPLE 2. *Find log* 236·8

The mantissa of log 236 = 3729.

Referring to the difference column at the extreme right of the Table in the same horizontal row as 236 and under 8, we find the number 15; this must be added to the four figures already obtained.

$$\therefore \log 236 = 3729$$
$$\text{diff. for } 8 = \underline{15}$$
$$\text{Hence,} \quad \log 236{\cdot}8 = \underline{2{\cdot}3744}$$

Similarly, $\log 0{\cdot}2368 = \bar{1}{\cdot}3744$

$$\log 0{\cdot}00002368 = \bar{5}{\cdot}3744$$
$$\log 236800 = 5{\cdot}3744.$$

EXAMPLE 3. *Find log* 0·004365.

From Table II., $\log 436 = 6395$

$$\text{diff. for } 5 = \underline{5}$$
$$\therefore \log 0{\cdot}004365 = \underline{\bar{3}{\cdot}6400}$$

The addition of the number from the difference columns to the number already found can usually be carried out mentally, and it is necessary in exceptional cases only to write down the numbers as shown.

The numbers in the difference columns from 10 to 20 are larger than in other parts of the table, hence, to ensure greater accuracy than would be possible with one line of differences, two lines are provided. The upper line refers to the numbers from 0 to 4, and the lower from 5 to 9.

To find the Number corresponding to a given Logarithm. The number corresponding to a given logarithm can be obtained by the converse of the preceding process, but the number is more easily and quickly obtained from a table of **antilogarithms**.

In proceeding to find the number corresponding to a given logarithm the mantissa only is considered. The characteristic merely determines the position of the decimal point

Referring to Table III. it will be seen that a series of numbers marked ·00 to ·99 are printed on the extreme left of the page, while the remaining part of the table is similar to the table of logarithms.

EXAMPLE 4. *Find the numbers whose logarithms are*

(i) 1·23, (ii) 2·235, (iii) 0·2357.

	0	1	2	3	4	5	6	7	8	9	Differences.		
											1 2 3	4 5 6	7 8 9
·23	**1698**	1702	1706	1710	1714	**1718**	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4

(i) From the table, a portion of which is shown here, it is seen that the number 1698 is immediately opposite and in the same row as 23. By the converse of the rule already stated, that the characteristic of a given logarithm determines the position of the decimal point, the number whose log is 1·23 is given by the table as 1698. Since the characteristic is 1, the required number is 16·98.

(ii) The number corresponding to row 23 and vertical column 5 is 1718. Since the characteristic is 2, the required number is 171·8.

(iii) The number corresponding to the first three figures is 1718. On adding the number 3, under 7 in the differences, we obtain 1721, and, since the characteristic is 0, the required number is 1·721.

EXAMPLE 5. *Find the number whose log is* 2·3567.

In the horizontal row with 35 and the vertical column with 6, the number 2270 is found. In the same row and in the difference columns on the right under the figure is the number 4. This difference must be added to the number previously obtained.

Thus the number is $2270 + 4 = 2274$.

Since the characteristic is 2, three figures must be marked off to the left of the decimal point. Hence, the number required is 227·4.

EXAMPLE 6. *Find the number whose log is* $\bar{2}$·3567.

Here the number, found as in the preceding example, is 2274. Since the characteristic is $\bar{2}$, it follows that one cipher precedes the first significant figure; hence the required number is 0·02274.

EXERCISES XXVI.

Find the log of

1. 75.	**2.** 36.	**3.** 99.
4. 7·5.	**5.** 3·6.	**6.** 0·0075.
7. 0·00036.	**8.** 756.	**9.** 0·0756.
10. 0·0000756.	**11.** 30·56.	**12.** 4·105.

Find the log of

13. (i) 2·307; (ii) 0·002307; (iii) 0·00002307.

14. (i) 37·24; (ii) 3·724; (iii) 0·003724.

Find the numbers corresponding to

15. $\log 2{\cdot}8537$. **16.** $\log 0{\cdot}0705$. **17.** $\log 1{\cdot}6862$.

18. $\log \bar{2}{\cdot}1933$. **19.** $\log 3{\cdot}9624$. **20.** $\log 0{\cdot}3795$.

21. $\log \bar{4}{\cdot}8061$. **22.** $\log \bar{1}{\cdot}5571$. **23.** $\log 3{\cdot}1019$.

24. $\log 4{\cdot}4324$. **25.** $\log 4{\cdot}6543$.

Multiplication by Logarithms.—Add the logarithms of the numbers together, and find the number corresponding to the sum of the logarithms; this is the product required.

EXAMPLE 1. *Multiply* 30·56 *by* 4·105.

From Table II., $\log 305 = 4843$
diff. for $6 = 9$

$\therefore \log 30{\cdot}56 = 1{\cdot}4852$

Similarly, $\log 4{\cdot}105 = 0{\cdot}6133$

$2{\cdot}0985$

$\text{antilog } 0{\cdot}098 = 1253$
diff. for $5 = 1$

$\therefore \text{antilog } 0{\cdot}0985 = 1254$

Hence, $30{\cdot}56 \times 4{\cdot}105 = 125{\cdot}4$.

EXAMPLE 2. *Multiply* 23·07 *by* 0·1354.

$\log 230 = 3617$
diff. for $7 = 13$

$\therefore \log 23{\cdot}07 = 1{\cdot}3630$

Similarly, $\log 0{\cdot}1354 = \bar{1}{\cdot}1316$

$0{\cdot}4946$

$\text{antilog } 494 = 3119$
diff. for $6 = 4$

3123

$\therefore 23{\cdot}07 \times 0{\cdot}1354 = 3{\cdot}123$.

In the process of adding the two logarithms together, the minus 1 in the lower will cancel the plus 1 in the upper line.

EXAMPLE 3. *Multiply* 0·2543 *by* 0·09027.

$$\begin{aligned} \log 254 &= 4048 \\ \text{diff. for } 3 &= \underline{\quad 5} \\ \therefore \log 0{\cdot}2543 &= \bar{1}{\cdot}4053 \\ \text{and } \log 0{\cdot}09027 &= \bar{2}{\cdot}9555 \\ & \overline{\bar{2}{\cdot}3608} \end{aligned}$$

$$\begin{aligned} \text{antilog } 360 &= 2291 \\ \text{diff. for } 8 &= \underline{\quad 4} \\ \text{antilog } {\cdot}3608 &= \underline{2295} \end{aligned}$$

$$\therefore\ 0{\cdot}2543 \times 0{\cdot}09027 = 0{\cdot}02295.$$

EXERCISES XXVII.

1. $51{\cdot}47 \times 20{\cdot}7$.
2. $619{\cdot}3 \times 0{\cdot}117$.
3. $30{\cdot}98 \times 0{\cdot}00258$.
4. $70{\cdot}93 \times 4{\cdot}051$.
5. $321{\cdot}4 \times 3{\cdot}063$.
6. $85{\cdot}61 \times 0{\cdot}07561$.
7. $0{\cdot}000215 \times 3490000$.
8. $3{\cdot}54 \times 0{\cdot}026 \times 1{\cdot}34$.
9. $1{\cdot}102 \times 27{\cdot}01 \times 5{\cdot}002$.
10. $4{\cdot}5 \times 5{\cdot}6 \times 0{\cdot}866$.

11. Find the numerical value of $a \times b$ when

(i) $a = 30{\cdot}56$, $b = 4{\cdot}105$. (ii) $a = 23{\cdot}07$, $b = 0{\cdot}1354$.

(iii) $a = 53{\cdot}56$, $b = 0{\cdot}9397$. (iv) $a = 8{\cdot}102$, $b = 35{\cdot}14$.

(v) $a = 3{\cdot}405$, $b = 9{\cdot}123$.

Division by Logarithms.—**From the logarithm of the dividend subtract the logarithm of the divisor; the antilogarithm of the remainder is the quotient.**

EXAMPLE 1. *Divide* 30·56 *by* 4·105.

$$\begin{aligned} \log 30{\cdot}56 &= 1{\cdot}4852 \\ \log 4{\cdot}105 &= \underline{0{\cdot}6133} \\ \text{remainder} &= 0{\cdot}8719 \end{aligned}$$

$$\begin{aligned} \text{antilog } 871 &= 7430 \\ \text{diff. for } 9 &= \underline{\quad 16} \\ & \overline{7446} \end{aligned}$$

$$\therefore\ 30{\cdot}56 \div 4{\cdot}105 = 7{\cdot}446.$$

The subtraction of the characteristics as a rule presents no difficulties. When, however, one characteristic or both are negative some amount of care is necessary to prevent mistakes.

EXAMPLE 2. *Divide* 254·3 *by* 0·09027.

$$\log 254{\cdot}3 = 2{\cdot}4053$$
$$\log 0{\cdot}09027 = \bar{2}{\cdot}9555$$
$$\text{remainder } 3{\cdot}4498$$

In subtracting the lower line from the upper, the minus sign of the characteristic is altered mentally to plus 2, and added to the upper characteristic.

Thus $2+2=4$ less 1 carried from the preceding figure, gives 3 as the result.

Referring to Table III., $\text{antilog } 449 = 2812$

$$\text{diff. for } 8 = \quad 5$$
$$\therefore \text{ antilog } 4498 = 2817$$
$$\therefore \ 254{\cdot}3 \div 0{\cdot}09027 = 2817.$$

EXAMPLE 3. *Divide* 0·09027 *by* 254·3.

$$\log 0{\cdot}09027 = \bar{2}{\cdot}9555$$
$$\log 254{\cdot}3 = 2{\cdot}4053$$
$$\bar{4}{\cdot}5502$$

In subtracting, the characteristic 2 in the lower line is altered mentally from +2 to −2, and added to the upper line, giving minus four as the result.

$$\text{antilog } 550 = 3548$$
$$\text{diff. for } 2 = \quad 2$$
$$\text{antilog } 5502 = 3550$$
$$\therefore \ 0{\cdot}09027 \div 254{\cdot}3 = 0{\cdot}000355.$$

EXERCISES XXVIII.

1. $360{\cdot}2 \div 898{\cdot}9$.
2. $15{\cdot}17 \div 347{\cdot}2$.
3. $907{\cdot}9 \div 17{\cdot}03$.
4. $1{\cdot}57 \div 3{\cdot}142$.
5. $(31{\cdot}5 \times 3{\cdot}142) \div 36$.
6. $(200 \times 12) \div 13 \times 3{\cdot}142$.
7. $0{\cdot}7854 \times 19{\cdot}5 \times 0{\cdot}5$.

Calculate the values of $a \times b$ and $a \div b$ when

8. $a = 1{\cdot}013$, $b = 3{\cdot}375$.
9. $a = 13{\cdot}75$, $b = 0{\cdot}0125$.
10. $a = 17{\cdot}25$, $b = 0{\cdot}0023$.
11. $a = 16{\cdot}25$, $b = 4{\cdot}35$.
12. $a = 0{\cdot}5$, $b = 0{\cdot}0065$.
13. $2307 \div 1{\cdot}354$.
14. $254{\cdot}3 \div 0{\cdot}09027$.

Find the numerical values of

15. $\dfrac{321{\cdot}4\times 3{\cdot}063}{70{\cdot}93\times 4{\cdot}051}$. 16. $\dfrac{30{\cdot}98\times 0{\cdot}00258}{51{\cdot}47\times 20{\cdot}7}$. 17. $\dfrac{3{\cdot}214\times 0{\cdot}7423}{7{\cdot}912}$.

Involution by Logarithms.—Multiply the logarithm of the number by the index of the power.

As $$10^{0{\cdot}4771}=3,$$
$$10^{0{\cdot}4771}\times 10^{0{\cdot}4771} \quad \text{or} \quad 10^{2\times 0{\cdot}4771}=3^2.$$

Example 1. *Find the value of* 3^2.

$$\log 3=0{\cdot}4771,$$
and $$0{\cdot}4771\times 2=0{\cdot}9542,$$
and this number corresponds to log 9.

Example 2. *Find the third power of* $4{\cdot}296$.

$$\log 4{\cdot}296=0{\cdot}6331$$
$$3$$
$$1{\cdot}8993$$
$$\text{antilog } 8993=7930\,;$$
$$\therefore\ 4{\cdot}296^3=79{\cdot}3.$$

Example 3. *Find the value of* $(32{\cdot}15)^{1{\cdot}52}$.

$$\log 32{\cdot}15=1{\cdot}5072.$$

The product of $1{\cdot}5072$ and $1{\cdot}52$ may be obtained by contracted multiplication

$$1{\cdot}5072\times 1{\cdot}52$$
$$251$$
$$15072$$
$$7536$$
$$301$$
$$2{\cdot}2909$$
$$\text{antilog } 2909=1954.$$

Since the characteristic is 2, the required number is 195·4.

Example 4. *Compute* $(2{\cdot}307)^{0{\cdot}65}$.

$$\log 2{\cdot}307=0{\cdot}3630,$$
and $$0{\cdot}363\times 0{\cdot}65=0{\cdot}2359.$$
$$\text{antilog } 2359=1722\,;$$
$$\therefore\ (2{\cdot}307)^{0{\cdot}65}=1{\cdot}722.$$

Evolution by Logarithms.—Divide the logarithm of the number by the index of the root; the antilogarithm of the quotient is the required answer.

EXAMPLE 5. *Find the cube root of* 8.

This problem may be written as $\sqrt[3]{8}$ or $8^{\frac{1}{3}}$.

Hence, the process of evolution reduces to that of involution: thus, to extract the cube root of 8 is the same as raising 8 to the power $\frac{1}{3}$.

$$\log 8 = 0{\cdot}9031,$$

and $$0{\cdot}9031 \div 3 = 0{\cdot}3010,$$

and this corresponds to 2:

$$\therefore \sqrt[3]{8} = 2.$$

EXAMPLE 6. *Evaluate* $\sqrt[3]{37{\cdot}24}$.

$$\log 37{\cdot}24 = 1{\cdot}5710,$$

and $$\frac{1{\cdot}5710}{3} = 0{\cdot}5236.$$

$$\text{antilog } 5236 = 3339;$$

$$\therefore \sqrt[3]{37{\cdot}24} = 3{\cdot}339.$$

EXAMPLE 7. *Find the square root of* 3·724.

$$\log 3{\cdot}724 = 0{\cdot}5710,$$

and $$\frac{0{\cdot}571}{2} = 0{\cdot}2855.$$

$$\text{antilog } 0{\cdot}2855 = 1{\cdot}93;$$

$$\therefore \sqrt[2]{3{\cdot}724} = 1{\cdot}93.$$

EXAMPLE 8. *Find the value of the cube root of the product of* 2·354 *and* 1·607.

$$\log 2{\cdot}354 = 0{\cdot}3718$$
$$\log 1{\cdot}607 = 0{\cdot}2060$$
$$\text{log of product} = 0{\cdot}5778$$

Also $$\frac{0{\cdot}5778}{3} = 0{\cdot}1926.$$

$$\text{antilog } 1926 = 1558;$$

$$\therefore \sqrt[3]{2{\cdot}354 \times 1{\cdot}607} = 1{\cdot}558.$$

EXAMPLE 9. *Find the value of the square root of the product of* 84·05 *and* 0·1357 *divided by* 1·163.

$$\begin{aligned} \log 84{\cdot}05 &= 1{\cdot}9246 \\ \log 0{\cdot}1357 &= \bar{1}{\cdot}1325 \\ & \quad \overline{1{\cdot}0571} \\ \log 1{\cdot}163 &= 0{\cdot}0656 \\ & \quad \overline{0{\cdot}9915} \end{aligned}$$

and $\frac{1}{2} \times 0{\cdot}9915 = 0{\cdot}4957.$

$\text{antilog } 0{\cdot}4957 = 3{\cdot}131\ ;$

$\therefore\ \sqrt{(84{\cdot}05 \times 0{\cdot}1357 \div 1{\cdot}163)} = 3{\cdot}131.$

EXERCISES XXIX.

1. $\sqrt[3]{(8{\cdot}742)}$.
2. $\sqrt[3]{37{\cdot}24}$.
3. $\sqrt{3{\cdot}724}$.
4. $\sqrt[3]{(2{\cdot}354 \times 1{\cdot}607)}$.
5. $\sqrt[3]{(27{\cdot}27 \times 0{\cdot}08765)}$.
6. Find the fifth root of $76{\cdot}52 \times 0{\cdot}254$.
7. $\sqrt{84{\cdot}05 \times 0{\cdot}1357 \div 1{\cdot}163}$.
8. $\sqrt[4]{(0{\cdot}086 \times 39{\cdot}86)}$.
9. $(1{\cdot}56)^{1{\cdot}5} \times (1{\cdot}5)^2 \times (1{\cdot}56)^{2{\cdot}5}$.
10. $\sqrt{\left(\dfrac{1{\cdot}265 \times 0{\cdot}00271}{2{\cdot}382 \times 10{\cdot}71}\right)}$.
11. Find the tenth power and the tenth root of 0·7.
12. $\sqrt{\dfrac{80}{3{\cdot}142}}$.
13. $\dfrac{8352 \times 3{\cdot}69}{(30{\cdot}57)^3}$.
14. $\dfrac{32 \times 100 \times 80 \times (1{\cdot}275)^2}{3{\cdot}142 \times (0{\cdot}25)^4}$.
15. $\sqrt[3]{15{\cdot}6} \times (0{\cdot}0045)^{\frac{3}{4}} \div (0{\cdot}00065)^{\frac{1}{2}}$.
16. $\sqrt{(120{\cdot}7 \times 19{\cdot}2 \times 40{\cdot}2 \times 61{\cdot}3)}$.
17. $\sqrt{(1{\cdot}945 \times 0{\cdot}195 \times 0{\cdot}915 \times 0{\cdot}835)}$.
18. $\sqrt[3]{\left(\dfrac{3 \times 8987}{2 \times 3{\cdot}142}\right)}$.
19. $\sqrt[3]{\left(\dfrac{1728 \times 0{\cdot}4764}{0{\cdot}5236}\right)}$.
20. $\sqrt{\dfrac{(2{\cdot}05)^2 \times 2{\cdot}24}{0{\cdot}0041}}$.
21. $\sqrt[3]{(0{\cdot}05)^3 \times (1{\cdot}04)^2}$.
22. $\sqrt{\dfrac{0{\cdot}076 \times 0{\cdot}0007}{0{\cdot}000019}}$.

Find the value of

23. $\dfrac{5{\cdot}473 \times 2{\cdot}517 \times 3{\cdot}5}{1{\cdot}324 \times 4{\cdot}768}$.

24. $\dfrac{3{\cdot}408 \div 6{\cdot}727 \div 2{\cdot}5}{5{\cdot}643 \times 1{\cdot}314}$.

25. Divide 8·306 times the cube of 27 by 0·0625 times the fourth power of 9.

Calculation of Logarithms.—The calculation of the numerical values of the logarithms of numbers is effected usually by what are called *series*, but values true to three or four decimal places are obtained readily by the use of squared paper, as follows:

The square root of 10 may be obtained by the usual process for finding the square root of a number, and is found to be 3·162.

Hence, $\sqrt{10} = 10^{0{\cdot}5} = 3{\cdot}162.$

If, now, we find the square root of 3·162, we obtain 1·7783, and this is the fourth root of 10;

$$\therefore\ 10^{\frac{1}{4}} = 10^{0{\cdot}25} = 1{\cdot}7783.$$

The square root of 1·7783 will be the eighth root of 10:

$$\therefore\ \sqrt[8]{10} = 10^{0{\cdot}125} = 1{\cdot}3336.$$

In a similar manner the square root of 1·3336 will be $10^{\frac{1}{16}}$, etc.

The results obtained may be tabulated as follows:

$$10^{\frac{1}{16}} = 10^{0{\cdot}0625} = 1{\cdot}1547.$$

$$10^{\frac{1}{8}} = 10^{0{\cdot}125} = 1{\cdot}3336.$$

$$10^{\frac{1}{4}} = 10^{0{\cdot}25} = 1{\cdot}7783.$$

$$10^{\frac{3}{8}} = 10^{0{\cdot}375} = 2{\cdot}3715.$$

The last result is obtained by multiplying $10^{\frac{1}{4}}$ by $10^{\frac{1}{8}}$:

$$\therefore\ 1{\cdot}7783 \times 1{\cdot}3336 = 2{\cdot}3715.$$

It is obvious, on examining the results obtained, that 10 raised to some power between 0·25 and 0·375 will give 2.

To find this power, plot the indices 0·125, 0·25 and 0·375 and the corresponding numbers 1·334, 1·778 and 2·371 respectively on squared paper as in Fig. 56. Then, corresponding to the point 2, the index 0·301 is found, hence, $10^{0·301} = 2$.

Again, multiplying 2·371 by 1·778 or $10^{\frac{1}{4}} \times 10^{\frac{3}{8}}$ we obtain $10^{\frac{5}{8}} = 10^{0·625} = 4·2172$.

On plotting the indices 0·375, 0·5 and 0·625 and the corresponding numbers 2·371, 3·162 and 4·217 respectively, and drawing a curve by means of a bent strip of wood or metal through the plotted points, we find, corresponding to the point 4, the number 0·602. Hence $10^{0·602} = 4$.

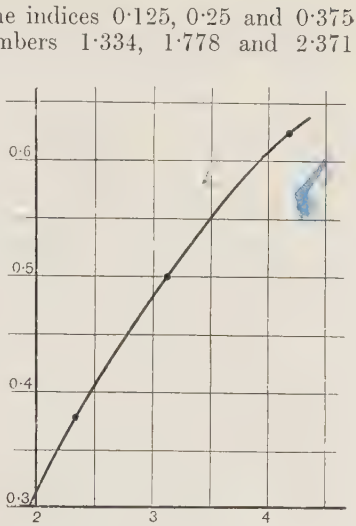


FIG. 56.—Logarithmic curve.

By repeated multiplication in this manner the logarithms of all the numbers 2, 3, 4, 5, 6, etc., may be found.

Simple and Compound Interest.—Questions dealing with interest may be solved by ordinary arithmetical methods (p. 95), but in many cases they are dealt with by logarithms much more easily, more especially questions on **compound interest.**

If P denotes the principal and r the rate per cent. per annum, then the simple interest for n years is given by

$$\text{Interest} = P \times \frac{r}{100} \times n = \frac{Prn}{100}.$$

Or, to obtain the simple interest on a loan for a number of years at a certain rate per cent.: **Multiply the principal by the rate per cent., and by the number of years, and divide the product by 100.**

The amount A at the end of n years will be the principal together with the interest, or

$$A = P + \frac{Prn}{100} = P\left(1 + \frac{rn}{100}\right).$$

For 1 year $n = 1$;

$$\therefore\ A = P\left(1 + \frac{r}{100}\right).$$

EXAMPLE 1. *Find the interest on £200 for 3 years at 5 per cent. (a) simple interest; (b) compound interest.*

(*a*) Simple interest for 3 years at 5 per cent $= 200 \cdot \frac{5}{100} \cdot 3 = £30.$

(*b*) Interest for one year $= 200 \times \frac{5}{100} = £10.$

Principal at end of first year $= £210.$

Interest for second year $= \frac{210 \times 5}{100} = £10{\cdot}5.$

Principal at end of second year $= 210 + 10{\cdot}5 = £220{\cdot}5.$

Interest for third year $= 220{\cdot}5 \cdot \frac{5}{100} = £11{\cdot}025.$

Principal at end of third year $= £220{\cdot}5 + £11{\cdot}025 = £231{\cdot}525.$

Hence, compound interest $= £31{\cdot}525 = £31.\ 10s.\ 6d.$

This arithmetical method is tedious, even for the short period of time given, and such methods become practically useless for longer periods, such as 10 or more years. The results are readily obtainable to a fair degree of accuracy by four-figure logarithms, though five- or seven-figure tables are preferable.

If a sum of money P is lent at compound interest for n years at the rate of r per cent. per annum, then the amount A is given by the formula,

$$A = P\left(1 + \frac{r}{100}\right)^n. \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

In Example 1, $P = £200$, $n = 3$ years, and $r = 5$. Substituting these values,

$$A = 200\left(1 + \tfrac{5}{100}\right)^3$$
$$= 200(1{\cdot}05)^3 = £231{\cdot}5.$$
$$3 \log 1{\cdot}05 = 0{\cdot}0636$$
$$\log 200 = 2{\cdot}3010$$
$$2{\cdot}3646$$
$$\text{antilog } 2{\cdot}3646 = 231{\cdot}5.$$

Hence, the interest is £231. 10*s*. − £200 = £31. 10*s*.

Formula (i) may be proved as follows:

Compound Interest. The amount at the end of 1 year is given by

$$A_1 = P\left(1 + \frac{r}{100}\right). \quad \text{..... (i)}$$

The amount A_1 becomes the principal for the second year, hence,

$$A_2 = A_1\left(1 + \frac{r}{100}\right). \quad \text{..... (ii)}$$

But from (i) $$A_1 = P\left(1 + \frac{r}{100}\right).$$

Substituting this value in (ii)

$$A_2 = P\left(1 + \frac{r}{100}\right)^2.$$

Similarly, for the third year the amount would be

$$A_3 = P\left(1 + \frac{r}{100}\right)^3,$$

and for n years $$A = P\left(1 + \frac{r}{100}\right)^n \quad \text{.....(iii)}$$

EXAMPLE 2. *Find the simple interest on* £450 *for* 7 *years* 3 *months at* $3\frac{1}{2}$ *per cent.*

As 7 years 3 months = 7·25 years.

$$\text{Simple interest} = 450 \times \frac{3{\cdot}5}{100} \times 7{\cdot}25$$

$$= 450 \times 0{\cdot}035 \times 7{\cdot}25$$

$$= £114.\ 4s.$$

$$\log 450 = 2{\cdot}6532$$
$$\log 0{\cdot}035 = \bar{2}{\cdot}5441$$
$$\log 7{\cdot}25 = 0{\cdot}8603$$
$$2{\cdot}0576$$

$$\text{antilog } 2{\cdot}0576 = 114{\cdot}2.$$

In the formula for simple interest

$$I = P \times \frac{r}{100} \times n = \frac{Prn}{100},$$

Of the four terms I the interest, P the principal, r the rate, and n the number of years, any three being given, the remaining term may be found.

EXAMPLE 3. *At what rate per cent. per annum simple interest will* £640 *amount to* £768. 18*s.* *in* 5 *years* 9 *months?*

5 years 9 months = 5·75 years.

Interest is £768. 18*s.* − £640 = £128. 18*s.*

= £128·9.

Let r denote the rate per cent., then, substituting these values in the equation given above,

$$128{\cdot}9 = 640 \times \frac{r}{100} \times 5{\cdot}75$$

$$6{\cdot}4 \times r \times 5{\cdot}75,$$

$$\therefore \quad r = \frac{128{\cdot}9}{6{\cdot}4 \times 5{\cdot}75} = 3{\cdot}5 \text{ per cent.}$$

$\log 128{\cdot}9 = 2{\cdot}1103$	$\log 6{\cdot}4 = 0{\cdot}8062$
$1{\cdot}5659$	$\log 5{\cdot}75 = 0{\cdot}7597$
$0{\cdot}5444$	$1{\cdot}5659$

antilog $0{\cdot}5444 = 3{\cdot}502$.

EXAMPLE 4. *In what time will* £3400 *amount to* £4709 *at* $5\frac{1}{4}$ *per cent. simple interest?*

Interest = £4709 − £3400 = £1309.

Substituting these values in our equation,

$$1309 = 3400 \times \frac{5{\cdot}25}{100} \times n,$$

where n denotes the number of years.

$$n = \frac{1309}{34 \times 5{\cdot}25} = 7\tfrac{1}{8} \text{ years.}$$

$\log 1309 = 3{\cdot}1169$	$\log 34 = 1{\cdot}5315$
$2{\cdot}2517$	$\log 5{\cdot}25 = 0{\cdot}7202$
$0{\cdot}8652$	$2{\cdot}2517$

antilog $0{\cdot}8652 = 7{\cdot}331$.

EXAMPLE 5. *Draw a graph from which the amount at the end of 5 or a smaller number of years of a sum of £200 at 5 per cent. simple interest can be obtained.*

The interest for 1 year is

$$200 \times \tfrac{5}{100} = £10,$$

and for 5 years is £50.

Set off the number of years on OX; then the vertical scale may be as marked in Fig. 57. Mark the point denoting 250 on the vertical axis and 5 on the horizontal axis as A. Join A to O; then the amount for any intermediate year may be obtained by inspection.

If the line OA be produced the *amounts* for a greater number than 5 years can be found.

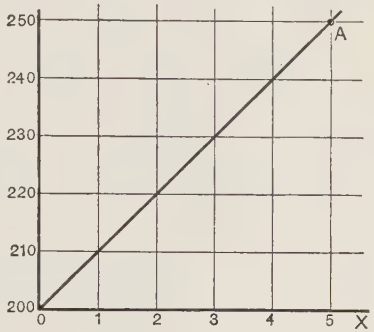


FIG. 57.

EXAMPLE 6. *Find the difference between (a) the simple interest, (b) the compound interest on £250 for 12 years at 3 per cent. per annum.*

(*a*) Simple interest $= 250 \times \tfrac{3}{100} \times 12 = £90.$

(*b*) $A = 250(1{\cdot}03)^{12} = £356$;

$$\therefore \text{ Interest} = £356 - £250 = £106.$$

$$\text{Difference} = £106 - £90 = £16.$$

EXAMPLE 7. *A certain sum of money, lent for 17 years at $3\frac{1}{2}$ per cent. per annum compound interest, amounted to £459 at the end of that period. Find the sum lent.*

Let P denote the sum, then as A is £459;

$$\therefore \quad 459 = P\left(1 + \frac{3{\cdot}5}{100}\right)^{17} = P(1{\cdot}035)^{17},$$

$$P = \frac{459}{(1{\cdot}035)^{17}} = £256{\cdot}2 \text{ or } £256.\ 4s.$$

$$\begin{aligned} \log 459 &= 2{\cdot}6618 \\ 17 \log 1{\cdot}035 &= 0{\cdot}2533 \\ &\underline{\quad\quad} \\ &2{\cdot}4085 \end{aligned} \qquad \text{antilog } 2{\cdot}4085 = 256{\cdot}2.$$

EXAMPLE 8. *Find the amount of £200 in 12 years at 4 per cent. compound interest.*

Here $P=200$, $r=4$ and $n=12$. Hence, substituting in (iii),

$$A=200\left(1+\tfrac{4}{100}\right)^{12},$$

$$\text{or } A=200\times(1{\cdot}04)^{12}=£319.\ 18s.$$

$12\log 1{\cdot}04=0{\cdot}2040$
$\log 200=2{\cdot}3010$
$2{\cdot}5050$ antilog $2{\cdot}5050=319{\cdot}9$.

EXAMPLE 9. *£150 is invested at 3 per cent. compound interest. At the end of how many years will the amount first exceed £200?*

Here we must find the number of years required for £150 to become £200, and the equation is

$$200=150(1{\cdot}03)^n,$$

where n denotes the number of years required.

$$(1{\cdot}03)^n=\frac{200}{150}=\frac{4}{3}=1{\cdot}333\ ;$$

$$\therefore\ n=\frac{\log 1{\cdot}333}{\log 1{\cdot}03}=\frac{0{\cdot}1249}{0{\cdot}0128}=9{\cdot}759 \text{ years.}$$

Discount and Present Worth.—**Discount** is the term applied to a percentage, or abatement, of the value of goods allowed by the seller for a cash payment.

EXAMPLE 1. *Find the cost of 30 yds. of material at 2s. 6d. per yd., allowing 5 % discount for cash.*

$$\text{Cost of 30 yds.}=30\times 2{\cdot}5s.$$

$$\text{Cost at 5 \% discount}=30\times 2\tfrac{1}{2}\times\tfrac{95}{100}$$

$$\text{or } 30\times 2\tfrac{1}{2}-\tfrac{5}{100}(30\times 2\tfrac{1}{2})$$

$$=71\tfrac{1}{4}s.=£3.\ 11s.\ 3d.$$

When a sum of money is paid before it is due, an abatement called **discount** is also made. Thus, a sum of £200 lent for one year at 5 per cent. will amount to £205; hence the present value, or **present worth** of £205 due at the end of a year is £200.

This sum is called **true discount**, to distinguish it from its commonly calculated value, or **banker's discount**, which is greater than the true discount. In other words, when a bill is discounted at 5 per cent., the interest charged is slightly greater than 5 per cent.

EXAMPLE 2. *What is the present value of £205 due 12 months hence at 5 per cent.?*

$$\text{Interest} = \frac{205 \times 1 \times 5}{100} = £10.\ 5s.$$

Amount of £205 for 1 year at $5\% = £215\frac{1}{4}$.

Present worth of $£215\frac{1}{4}$ is £205.

" " £1 is $\frac{205}{215\frac{1}{4}}$.

" " £205 is $\frac{205 \times 205}{215\frac{1}{4}}$

$$= \frac{205^2 \times 4}{861} = £195.\ 6s.$$

$2 \log 205 = 4{\cdot}6236$
$\log 4 = 0{\cdot}6021$
$5{\cdot}2257$
$\log 861 = 2{\cdot}9350$ antilog $2{\cdot}2907 = 195{\cdot}3$.
$2{\cdot}2907$

EXAMPLE 3. *How much will a broker who charges 5 per cent. discount give for a bill for £600 due seven months hence?*

$$\text{Interest} = 600 \times \tfrac{7}{12} \times \tfrac{5}{100} = £17.\ 10s.$$

$$\text{Amount} = £617{\cdot}5\ ;$$

$$\therefore \text{ present value} = \frac{600^2}{617{\cdot}5} = £583.$$

$2 \log 600 = 5{\cdot}5564$
$\log 617{\cdot}5 = 2{\cdot}7907$ antilog $2{\cdot}7657 = 583$.
$2{\cdot}7657$

EXAMPLE 4. *What is the present value of £399 payable 4 years hence at $3\frac{1}{2}$ per cent.?*

$$\text{Interest} = 399 \times 4 \times \frac{3{\cdot}5}{100} = £55{\cdot}88.$$

$\log 3{\cdot}99 = 0{\cdot}6010$
$\log 4 = 0{\cdot}6021$
$\log 3{\cdot}5 = 0{\cdot}5441$ antilog $1{\cdot}7472 = £55{\cdot}88$.
$1{\cdot}7472$

$$\text{Amount} = 399 + 55{\cdot}88 = £454{\cdot}88.$$

$$\text{Present worth} = \frac{399^2}{454{\cdot}88} = \frac{399^2}{8 \times 56{\cdot}86} = £350.$$

$2 \log 399 = 5{\cdot}2020$ $\log 8 = 0{\cdot}9031$
$2{\cdot}6579$ $\log 56{\cdot}86 = 1{\cdot}7548$ antilog $2{\cdot}5441 = 350$.
$2{\cdot}5441$ $2{\cdot}6579$

EXERCISES XXX.

Find the simple interest on

1. £800. 15*s.* for 4 years at 5 %.

2. £888 for 4½ years at 3¼ per cent. per annum.

3. £270 for 5 years at 4½ per cent. per annum.

4. £325. 16*s.* 8*d.* for 4 years and 3 months at 3½ per cent.

5. A bankrupt owing £4000 pays his creditors at the rate of 13*s.* 4*d.* in the pound. If this sum of money is allowed to accumulate at 5 per cent. simple interest, in how many years will it reach the amount of the original debt?

6. If the simple interest on £4373. 6*s.* 8*d.* for 1½ years is £246. what is the rate per cent.?

7. In what time will £1675 amount to £1995. 6*s.* 10½*d.* at 4½ per cent. per annum simple interest?

8. What principal will amount at simple interest to £420 in 3 years at 4 per cent.?

9. At what rate per cent. will £540 amount to £734. 8*s.* in 9 years?

10. What sum of money at 5 per cent. for 4 years and 6 months will amount to £5145. 12*s.* 3*d.*?

11. What is the rate of interest (simple) on £540 if it amounts to £603 in 3 years 4 months?

12. Find the present worth of £2000 due 3 years hence at 5 per cent.

13. Find the present worth of £1368. 8*s.* due 8 months hence at 5½ per cent. per annum.

14. Find the present value of £194. 7*s.* 6*d.* due 11 months hence at 4 per cent.

15. What is the present worth of £318. 19*s.* 9*d.* due in 7 months at 2⅖ per cent. per annum?

16. Find the discount on £402. 9*s.* 3*d.* for 25 days at 3½ per cent.

17. What is the present value of £399 payable 4 years hence at 3½ per cent.?

18. Find the present worth of £1593. 15*s.* due 15 months hence at 5 per cent.

19. A man has £350 in a bank at 2½ per cent. What will be the amount at the end of 6 years? If the interest is added to the principal and bears interest, what will be the amount at the end of 6 years?

20. Find the amount of £1800 for 3 years at $4\frac{1}{2}$ per cent. compound interest.

21. In how many years will £100 amount to £150 at $2\frac{1}{2}$ % compound interest? Verify that your answer is nearly equal to $40{\cdot}55/r+0{\cdot}22$, where $r=2\frac{1}{2}$.

Find the amount at compound interest of

22. £200 in 3 years at 4 per cent.

23. £265 in 3 years at 5 per cent.

24. £1205 in 2 years at 5 per cent.

25. £2000 in 3 years 3 months at 4 per cent.

26. £100 in 4 years 9 months at $3\frac{1}{2}$ per cent.

27. £400 in 2 years 6 months at 4 per cent.

28. Find the difference between the simple and compound interest on £635. 10*s*. for 3 years 9 months at 3 per cent. per annum.

29. Find the compound interest on 100 guineas for 3 years at $4\frac{1}{4}$ per cent. per annum.

Stocks and Shares.—Two forms of investment for money consist either in purchasing what is called Government or guaranteed Stock, or in shares in a business company. British Government Stock, which constitutes what is called the National Debt, was at one time mainly at 3 per cent. and known as 3 per cent. **consols**. The interest which is payable yearly, half yearly, or at definite intervals, is not necessarily 3 per cent. but may vary from time to time.

In a business company the profits are divided among the shareholders yearly, or half-yearly, in the proportion of the number of **shares**. The money earned by each share is called the **dividend**.

More than they originally cost has to be paid for shares in a successful company which pays a high dividend, and less than the original cost when the dividend is low.

Thus, for example, if a company is able to pay 8, 10 or more, per cent., then shares which at first cost £100 may be worth £120 or more; in the latter case those holding shares and wishing to sell may have to accept less than their original cost. Thus, shares which cost £100 may have to sell for £80, £70 or less.

In the former case the shares are said to be above and in the latter below **par**. When £100 of shares sells for £100 the shares are said to be at **par**.

The capital or **stock** of a company is of different kinds and receives special names; thus **debentures** and **debenture stock** are stocks which bear a fixed rate of interest and also offer good security for the return of the principal.

Preference stock bears a **fixed rate** of interest which must be paid before the dividend can be paid on the **ordinary stock**. Sales of stock are usually carried out by a **stockbroker** who charges a **brokerage** varying from $\frac{1}{8}$ to 1 per cent.

EXAMPLE 1. *How much money must be invested in* 3 *per cent. stock at* $98\frac{1}{4}$ *to yield an income of* £120?

To obtain £3 a year £$98\frac{1}{4}$ must be invested.

,, £1 ,, $\frac{£98\frac{1}{4}}{3}$,, ,,

,, £120 ,, $\frac{£98\frac{1}{4} \times 120}{3}$,, ,,

$= £3930.$

EXAMPLE 2. *What will be the annual income obtained from* £3930 *invested in* 4 *per cent. stock at* $127\frac{1}{8}$?

£$127\frac{1}{8}$ gives an income of £4;

$\therefore$ £1 ,, ,, $\frac{£4}{127\frac{1}{8}}$;

$\therefore$ £3930 ,, ,, $\frac{£4 \times 3930}{127\frac{1}{8}}$

$= £123.\ 13s.\ 2d.$

EXAMPLE 3. *A sum of* £1000 *was invested in the purchase of* $2\frac{1}{2}$ *per cent. stock at* $88\frac{3}{4}$. *Find how much stock was obtained and the annual income that will be received. What would have been the income if the* £1000 *had been invested in a* $3\frac{1}{2}$ *per cent. stock at* 104?

Amount of stock obtained $= £1000 \times \frac{100}{88\frac{3}{4}}$

$= £1126{\cdot}76 = £1126.\ 15s.\ 2d.$

Annual income received $= \frac{£1000 \times 2\frac{1}{2}}{88{\cdot}75}$

$= £28{\cdot}17 = £28.\ 3s.\ 4{\cdot}8d.$

Annual income from $3\frac{1}{2}$ per cent. stock

$$= \frac{£1000 \times 3{\cdot}5}{104} = £33{\cdot}66$$

$$= £33.\ 13s.\ 1d.$$

The following cutting is from a daily newspaper, and gives some money market prices for railways and banks on Nov. 9, 1907.

BRITISH RAILWAYS.

Caledonian	89	London and Brighton Deferred	$77\frac{1}{4}$
Caledonian Deferred	25	London and N.-W.	$138\frac{3}{8}$
Great Eastern	$66\frac{1}{2}$	London and S.-W.	134
Great Northern Pref.	$91\frac{1}{2}$	Midland Pref.	$62\frac{1}{4}$
Great Western	$120\frac{1}{4}$	Midland Deferred	$60\frac{1}{2}$
Lancashire and Yorkshire	$95\frac{1}{4}$	North-Eastern	$133\frac{1}{4}$
London and Brighton	99		

BANKS.

London and S.-W.	71	London and Westminster	$50\frac{1}{4}$
London City and Midland	47	Parrs'	$82\frac{1}{2}$
London and County	$89\frac{1}{4}$		

EXAMPLE 4. *How much Great Western stock at* $120\frac{1}{4}$ *can be purchased for* £2000?

£$120\frac{1}{4}$ buys £100 stock;

$$\therefore \text{ £1 buys } \frac{100}{120\frac{1}{4}};$$

$$\text{and £2000 buys } \frac{2000 \times 100}{120\frac{1}{4}} = £1663.\ 4s.$$

EXAMPLE 5. *What annual income will be obtained by investing* £500 *in the London and South-Western* 4 *per cent. stock at* 134?

For £134 an income of £4 is obtained;

$$\therefore \text{ income for £500} = \frac{500 \times 4}{134}$$

$$= £14.\ 18s.\ 6d.$$

EXAMPLE 6. *What would be the income from* £3500 *invested in London and South-Western Bank shares, paying* 64*s. on each* £20 *fully paid up?*

64*s.* on 400*s.* is 16 per cent.;

$$\therefore \text{ income} = \frac{3500 \times 16}{100} = £560.$$

EXAMPLE 7. *Which would be the better stock to invest in, the $3\frac{1}{2}$ per cents. at 106, or the 5 per cents. at $153\frac{1}{2}$?*

In the first stock £3·5 is obtained by investing £106:

$$\therefore \text{ £1 is obtained by investing } \frac{106}{3{\cdot}5} = 30{\cdot}3.$$

In the second case £5 is obtained by investing $153\frac{1}{2}$;

$$\therefore \text{ £1 is obtained by investing } \frac{153{\cdot}5}{5} = 30{\cdot}7.$$

Hence, the first-named stock is the better.

EXAMPLE 8. *Which would yield the better return for an investment, Great Western stock at $120\frac{1}{4}$, or London and North-Western stock at $138\frac{1}{2}$ if the former pays $5\frac{1}{8}$ per cent. and the latter $5\frac{3}{8}$ per cent.?*

£1 invested in the first-named stock yields $\frac{£5\frac{1}{8}}{120\frac{1}{4}} = £0{\cdot}0426.$

£1 invested in the second stock yields $\frac{£5\frac{3}{8}}{138\frac{1}{2}} = £0{\cdot}0406.$

Hence, the former stock is the better investment.

EXERCISES XXXI.

1. What price should be paid for 6 per cent. stock to give 4 per cent. on money invested in it?

2. Which would be the better investment, the 3 per cents. at 104 or the 5 per cents. at 150?

3. How much is obtained from the sale of £1400 stock of $2\frac{1}{2}$ per cent. at 105?

4. How much $3\frac{1}{2}$ per cent. stock at 97 can be purchased for £1800. 10*s.*, and what will be the dividend?

5. How much 3 per cent. stock has been sold out at 88, if, when the proceeds are invested in the 4 per cents. at 120, the income is reduced by £2 per year?

6. Which is the better investment, 3 per cent. stock at 87 or 6 per cent. stock at 168? What difference of annual income will it make whether a sum of £541. 6*s.* 8*d.* be invested in the one or the other of these stocks?

7. A person buys 5 per cent. stock at 40, and sells out at the end of the year when the stock has fallen to 38. How much per cent. does he lose on the transaction?

8. A person sells out £3965 from 3 per cent. stock at 74, and reinvests in $5\frac{1}{2}$ per cent. at 143. What is his gain or loss in annual income?

9. What interest per cent. does a man get for his money by investing in three per cent. stock at 90?

How much must he invest to give him an income of £1153 a year?

10. What is the rate per cent. of the interest that a person gets on money invested in a 4 per cent. stock, the price of which is $119\frac{3}{8}$ (including brokerage)? What annual income would he get on £1500 so invested?

11. If £100 of $2\frac{3}{4}$ per cent. stock cost £$97\frac{5}{8}$, what amount of the stock would cost £7497. 12*s*. 0*d*. and what annual income would be derived from it?

12. A man invests a fourth of his capital in $2\frac{1}{4}$ per cent. stock at 90, and the remainder in $3\frac{1}{2}$ per cent. stock at 105; find the average rate per cent. return on his capital.

13. A man invests £2000 in a 4 per cent. guaranteed stock when the price of £100 of stock was £$97\frac{5}{8}$; he sells out when the price of £100 of stock has risen to £154, and invests the proceeds in a $2\frac{1}{2}$ per cent. stock at £98. Neglecting the commissions, find whether his income is increased or diminished.

14. A man invests £3600 in 3 per cent. stock at 90. He sells out at 80, and lends $\frac{5}{8}$ of his money at 4 per cent., and $\frac{3}{8}$ at 5 per cent. How long must the loan last, so that when he reinvests his money in 3 per cents. at 90, his gain on interest (simple) may exactly equal his loss upon principal?

15. The capital of a trading company consists of 4000 *A* shares of £80 each and 2000 *B* shares of £25 each; in dividing the profits, 5 per cent. of the amount of each share is first paid, and then the remainder, if any, is divided equally amongst the shareholders. The profits of the undertaking in one year were £34853; how much would be paid to the holder of an *A* share and how much to the holder of a *B* share?

CHAPTER VI.

Area of Rectangle, Parallelogram: Practical Applications. Addition and Subtraction of Areas. Area of a Triangle, Equilateral Triangle, Hexagon, Trapezium, Quadrilateral.

Area.—The area of a surface (*i.e.* the amount of surface included between the bounding lines of the figure) is measured by the number of square units it contains.

The **dimensions** of a rectangle are its length and breadth, and since the area as already indicated, p. 39, is the product of the number of units in its length by the number of units in its breadth (or width) both expressed in the same units, it follows that:

Length of a rectangle = area ÷ breadth.

Breadth of a rectangle = area ÷ length.

When the two adjacent sides of a rectangle are equal in length, then the surface is bounded by four equal sides and is called a **square.**

EXAMPLE 1. *A rectangular field of 32 acres is one-tenth of a mile in length. What is its width?*

$$\tfrac{1}{10} \text{ mile} = 1760 \div 10 = 176 \text{ yds.}$$

$$32 \text{ acres} = 32 \times 4840 \text{ sq. yds.}; \quad \therefore \text{ width} = \frac{32 \times 4840}{176} = 880 \text{ yds.}$$

EXAMPLE 2. *What is the length in metres and in feet of a square room of 40 square metres area?*

Let l denote the length of a side. Then

$$l = \sqrt{40} = 6{\cdot}324 \text{ m.}$$

$$\tfrac{1}{2} \log 40 = \tfrac{1}{2}(1{\cdot}6021) = 0{\cdot}8010$$

$$\text{antilog } 0{\cdot}8010 = 6{\cdot}324$$

$$1 \text{ metre} = 3{\cdot}28 \text{ ft.}$$

$$\therefore \ 6{\cdot}324 \text{ metres} = 6{\cdot}324 \times 3{\cdot}28 = 20{\cdot}74 \text{ ft.}$$

Parallelogram.—Let $ABCD$ be a given parallelogram (p. 32). At the extremities of the base, A and B, erect perpendiculars meeting CD in E, and CD produced in F.

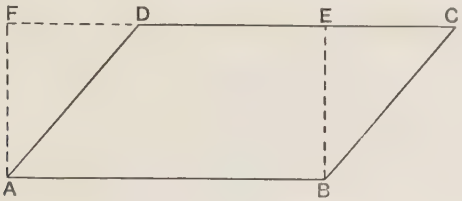


Fig. 58.—Area of a parallelogram.

The figure should be drawn on squared paper; then, if the triangle BEC be cut out and placed on the triangle AFD, so that BC coincides with AD, then E will coincide with F. Hence, the parallelogram $ABCD$ is equal to the rectangle $ABEF$.

If l denotes the length of the base, and h the height, or altitude, then the area of a parallelogram is given by

$$\text{Area} = lh.$$

Some Practical Applications.—The painting, papering, or distempering of the walls, or ceiling, of a room is usually estimated by the square yard, or the square foot.

Wall paper is usually sold by the piece or roll, 12 yards long. A paperhanger charges at the rate of so much per piece for placing it on the surface to be covered. Deductions from wall surfaces have to be made for doors, windows, etc., and allowance has to be made for the waste of paper in matching the pattern.

Example 3. *What would be the cost of painting the four walls of a room length 5 yds., breadth 4 yds. and height 3 yds., at 2s. per sq. yd.? Also find the cost of distempering the ceiling at 1s. per sq. yd.*

Take a square on squared paper to denote one square yard, place the four rectangles representing the walls together as in Fig. 59. They are obviously all of the same height and together form a rectangle.

$AEFK$ is made up of four rectangles representing the two sides and the two end walls respectively.

A model of the room is obtained by cutting out the figure and bending the paper along the lines BJ, HC and GD, so that EF coincides with AK.

FIG. 59.

The area may be obtained either by counting the enclosed squares or by calculation.

The area will be twice the area of the rectangle $AJHK$ together with twice the area of $BCHJ$

$$= 2AK(AB + BC) \text{ or } 2h(l + b).$$

The length AE, or twice the sum of the length and breadth, is called the **perimeter**, hence, the surface is the product of the perimeter and the height.

Using the given values:

$$\text{Area of walls} = 2 \times 3(5+4) = 54 \text{ sq. yds.},$$
$$\text{cost} = 54 \times 2s. = £5.\ 8s.$$
$$\text{Area of ceiling} = 5 \times 4 = 20 \text{ sq. yds.},$$
$$\text{cost} = 20 \times 1s. = £1.$$

EXAMPLE 4. *A room is* 18 *ft. long,* 16 *ft. wide and* 12 *ft. high. Allowing* 40 *sq. ft. for doors and windows, what will be the cost of papering the walls of the room with paper at* 2*s.* 4*d. per roll of* 12 *yds. and* 21 *in. wide?*

$$\text{Area of the four walls} = (2 \times 18 \times 12) + (2 \times 16 \times 12)$$
$$= 24(18 + 16) = 816 \text{ sq. ft.}$$

Deducting the area of doors and windows, this area becomes

$$816 - 40 = 776 \text{ sq. ft.}$$

The area of each roll of paper is $36 \times \frac{7}{4}$ sq. ft.,

and its cost is $2s.\ 4d. = \frac{7}{3}s.$;

$$\therefore \text{ cost of paper} = \frac{776 \times \frac{7}{3}}{36 \times \frac{7}{4}} = \frac{776 \times 7}{27 \times 7}.$$

$$\log 776 = 2{\cdot}8899$$
$$\log\ 27 = 1{\cdot}4314$$
$$\overline{1{\cdot}4585}$$
$$\text{antilog } 1{\cdot}4585 = 28{\cdot}74;$$
$$\therefore \text{ cost} = £1.\ 8s.\ 9d.$$

Carpeting.—A carpet is frequently made in one long continuous piece of a known width; hence, the length necessary to cover the floor of a room is found by dividing the area of the floor by the width of the carpet. As in papering, some allowance has usually to be made for waste in cutting.

EXAMPLE 5. *The floor of a room is* 24 *ft. long, and* 18 *ft. wide. Find the length and cost of a carpet* $\frac{3}{4}$ *yd. wide to cover it at* 5*s.* 4*d. per yd.*

As 24 ft. = 8 yds. and 18 ft. = 6 yds.;

$\therefore$ area of floor $= 8 \times 6 = 48$ sq. yds.

$$\text{Length of carpet} = \frac{48}{\frac{3}{4}} = 64 \text{ yds.}$$

$$\text{Cost} = 64 \times 5\tfrac{1}{3}s. = £17{\cdot}07$$

$$= £17.\ 1s.\ 4{\cdot}8d.$$

EXAMPLE 6. *The length of a room is* 18 *ft.* 9 *in., the width* 17 *ft.* 6 *in. Find the cost of a carpet* 27 *inches wide to cover it at* 5*s.* 3*d. per yard.*

18 ft. 9 in. = 18·75 ft.; 17 ft. 6 in. = 17·5 ft.;

$$\text{area of floor} = \frac{18{\cdot}75 \times 17{\cdot}5}{9} \text{ sq. yds.}$$

$$\text{Length required} = \frac{18{\cdot}75 \times 17{\cdot}5}{9 \times 0{\cdot}75}.$$

$$\text{Cost} = \frac{18{\cdot}75 \times 17{\cdot}5 \times 5\frac{1}{4}}{9 \times 0{\cdot}75 \times 20} = £12.\ 15s.\ 2\tfrac{1}{4}d.$$

$\log 18{\cdot}75 = 1{\cdot}2729$	$\log 9 = 0{\cdot}9542$
$\log 17{\cdot}5 = 1{\cdot}2430$	$\log 0{\cdot}75 = 1{\cdot}8751$
$\log 5{\cdot}25 = 0{\cdot}7202$	$\log 20 = 1{\cdot}3010$
$3{\cdot}2361$	$2{\cdot}1303$
$2{\cdot}1303$	
$1{\cdot}1058$	

antilog $1{\cdot}1058 = 12{\cdot}76$;

$\therefore$ cost $= £12{\cdot}76 = £12.\ 15s.\ 2\frac{1}{4}d.$

Linings.—The cost, or weight, of the lead or zinc used for lining the internal surface of a cistern, or tank, is obtained by calculating the area to be covered and multiplying by the cost, or weight, per square foot. The sheet lead, or zinc, used

for this and similar purposes would have a thickness represented by a small fraction of an inch, hence it is usually referred to in terms of its weight; thus, "6 lb. lead" denotes that one square foot of it weighs 6 lb.

EXAMPLE 7. *The internal dimensions of a rectangular tank without lid are as follows: length 6 ft., width 4 ft., depth 3 ft.*

If the inside of the tank is to be lined with 5 lb. lead, find the weight of lead required.

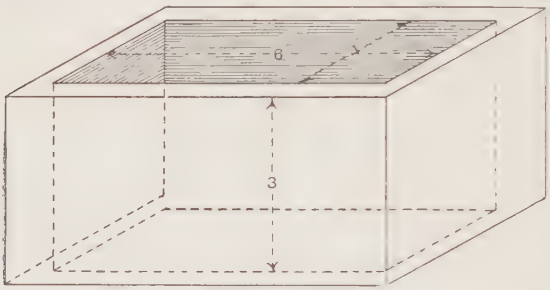


FIG. 60.

The area of the surface to be covered is found in the same manner as in the case of a wall surface (p. 127).

$\therefore$ area of the two sides $= 2 \times (6 \times 3) = 36$ sq. ft.

,, ,, ends $= 2 \times (4 \times 3) = 24$,,

area of base $6 \times 4 = 24$ sq. ft.;

total area to be covered $= 36 + 24 + 24 = 84$ sq. ft.;

weight of lead required $= 84 \times 5 = 420$ lb.

Addition and Subtraction of Areas.—A right-angled triangle may be used to find one side of a square, the area of which shall be equal to the sum or difference of the areas of two given squares.

EXAMPLE 8. *What is the side of a square, or the diameter of a circle, the area of which is equal to the sum of the areas of* (i) *two squares, side of one 3 in. of the other 4 in.; or* (ii) *two circles, diameters 3 in. and 4 in. respectively.*

Using the preceding construction (p. 33), the length of the side of a square or a diameter of a circle, is found to be 5 in.

EXAMPLE 9. *Find the side of a square the area of which shall be equal to the difference between the areas of a square 7 in. side and a square of 3 in. side.*

This problem is the converse of the preceding one. Thus, make $AC=3$ in., and at C erect a perpendicular. With A as centre, and a radius of 7 in., describe an arc of a circle intersecting CB at B. Then CB is the length required.

By calculation, $CB^2=7^2-3^2$,

or $(7+3)(7-3)=40$.

Therefore, $CB=\sqrt{40}=6{\cdot}3$.

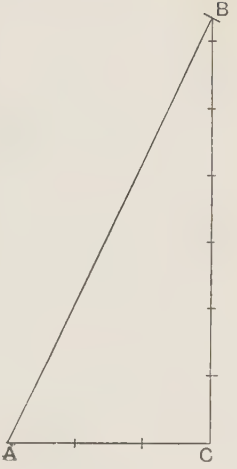


FIG. 61.

EXAMPLE 10. *The perimeter of a rectangular field is 800 yards. The length is three times the width. Find the area. What is the length of a diagonal of the field?*

As the perimeter of the rectangle is 800 yards, the sum of the two sides AB and BC, will be 400 yards. As the length is three times the width, AB will be 300 yds. and BC 100 yds.

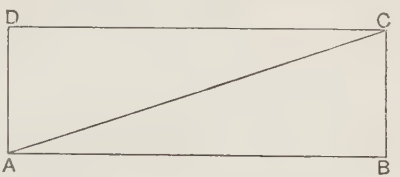


FIG. 62.

Area of

$ABCD=300\times100=30000$ sq. yds.

If on squared paper the side of one of the larger squares be taken to denote 100 yds., then the rectangle $ABCD$ may be drawn and the area ascertained by counting the squares. Also joining A to C the length AC will be found to be 316·2 yds.

By calculation, $AC^2=AB^2+BC^2$;

$$\therefore\ AC=\sqrt{300^2+100^2}=\sqrt{100000}$$
$$=316{\cdot}2 \text{ yds.}$$

$\frac{1}{2}\log 100000=\frac{1}{2}(5{\cdot}0000)=2{\cdot}50$,

antilog $2{\cdot}50=316{\cdot}2$.

EXERCISES XXXII.

1. The area of a rectangular field is 22 acres. If its length is ¼ mile, find the width in yards.

2. What will be the cost of putting a fence round a square field of 10 acres at 9*d.* per yard?

3. If 350 paving stones each 45 cm. wide and 50 cm. long are required to pave a courtyard 10½ metres long, what is its width?

4. A square field of 10 acres is enclosed by a barbed wire fence in which there are four rows of wire. What is the cost, if one ton of wire costs £14 and 100 yards of wire weigh 25 lb.?

5. Find the length of a side of a square field of 2½ acres and the cost of making a pathway immediately outside 6 feet wide at 9*d.* per square yard.

6. The diagonal of a rectangular plot is 53 ft., its length is 45 ft.: find the side of a square that shall equal this plot in area.

7. The areas of a square and a rectangular field are equal, the lengths of the sides of the latter being 135·5 and 103·8 metres respectively. Find the cost of enclosing the square field by a fence at 2·5 francs per metre.

8. A rectangular garden is to be surrounded by a wall 8 ft. 6 in. high and 9 in. thick. The inner dimensions of the garden being 65 ft. by 50 ft., how many bricks laid in mortar, the space per brick being 9 in. × 4½ in. × 3 in., are required to build the wall, and what will be the cost at £1. 5*s.* per thousand bricks?

9. The cost of levelling a square piece of ground at 7½*d.* per sq. ft. is £190. 2*s.* 6*d.* Find the cost of surrounding it with an iron railing at 2*s.* 7*d.* a yard.

10. The length of a rectangle is three times its width and the perimeter is 16·88 ft. Find the length of its sides and its area.

11. How many deal planks 15 ft. long by 9 in. wide are required to form a platform 10 yds. wide by 30 yds. long, allowing 10 planks for waste?

12. What would be the cost of painting one side of a wall 58·35 feet long and 17·75 feet high at 1*s.* 4*d.* per square foot?

13. The sides of a rectangle are measured and found to be 6·32 ft. and 5·68 ft. respectively. More careful measurements show that they are really 6·33 ft. and 5·69 ft. Find the area in each case, also find the percentage error.

14. Find the cost of papering a room 12 metres long, 6·2 metres broad and 4·25 metres high, with paper of which the breadth is 70 centimetres, and price is 2 francs per piece of 8 metres, 21 square metres being allowed for windows and doors.

15. A rectangular field is twice as long as it is broad, and it would cost £92. 8*s.* to put a fence round it at 3*s.* 6*d.* per yard of fence. Find the area of the field in acres.

16. A rectangular sheet of metal, initially 7·9 ft. and 5·3 ft. becomes when heated 7·93 ft. and 5·32 ft. Express the difference between the initial and final areas as a decimal of the original area.

17. A room is 7 metres long, $5\frac{1}{2}$ metres wide, and $3\frac{1}{2}$ metres high. What will be the cost of whitewashing the walls at 30 centimes per square metre? [£1 = 25 francs.]

Triangle.—If $ABCD$ is a parallelogram, the line AC (Fig. 63) will divide it into two equal parts. Hence, as the area of a parallelogram is the product of the length of the base and the altitude, the area of the triangle

$$= \tfrac{1}{2}(base \times altitude) = \tfrac{1}{2}ab.$$

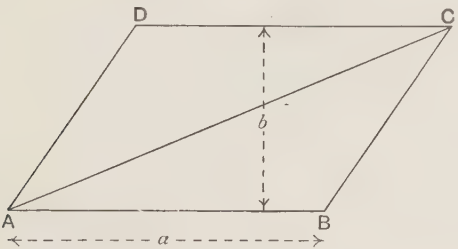


FIG. 63.—Area of a triangle.

The altitude, or height of a triangle denotes the perpendicular distance between the base and a line parallel to the base and passing through the vertex.

EXAMPLE 1. *In a right-angled triangle the two perpendicular sides are* 6 *and* 8 *in. respectively. Find the area.*

$$\text{Area} = \tfrac{1}{2}(6 \times 8) = 24 \text{ sq. in.}$$

The length of the third side or *hypotenuse* is $\sqrt{6^2 + 8^2} = 10$.

As the lengths of the three sides are known, the area may be found by the following rule:

From half the sum of the sides subtract the length of each side separately. Multiply the three remainders and the half sum together, then the square root of the product is the area required.

The rule may be written in a concise form by using the usual convention that the side CB opposite the angle A is denoted by a. Similarly, b and c denote the lengths of the sides opposite the angles B and C respectively.

$$s = \tfrac{1}{2}(a+b+c) = \tfrac{1}{2}(8+6+10) = 12,$$
$$s-a = 12-6 = 6,$$
$$s-b = 12-8 = 4,$$
$$s-c = 12-10 = 2.$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12\times 6\times 4\times 2}$$
$$= 24 \text{ sq. in.}$$

EXAMPLE 2. *The three sides of a triangular strip of ground are* 150, 200, *and* 250 *yards respectively. Find the cost of the strip at* £1 *per acre.*

$$s = \tfrac{1}{2}(150+200+250) = 300.$$
$$\text{Area} = \sqrt{300\times 150\times 100\times 50} = 15000 \text{ sq. yds.}$$

$$\log 300 = 2{\cdot}4771$$
$$\log 150 = 2{\cdot}1761$$
$$\log 5000 = 3{\cdot}6990 \qquad \text{antilog } 4{\cdot}1761 = 15000$$
$$2 \quad 8{\cdot}3522$$
$$4{\cdot}1761$$

$$\text{Cost} = \frac{15000}{4840}$$

$$\log 15000 = 4{\cdot}1761$$
$$\log 4840 = 3{\cdot}6848 \qquad \text{antilog } 0{\cdot}4913 = 3{\cdot}099.$$
$$0{\cdot}4913$$

$\therefore$ cost = £3. 2*s*.

Equilateral Triangle.—In an equilateral triangle, the three sides are all of equal length and the length of a line CD, perpendicular to AB (Fig. 64), can be found when the length of one side of the triangle is given. Hence the area of the triangle can be found. Or, as the lengths of the three sides are known, we may use the rule on p. 133.

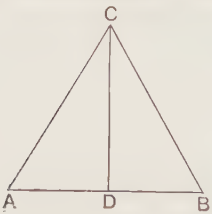


FIG. 64.—Equilateral triangle.

EXAMPLE 3. *Find the area of an equilateral triangle each side of which is* 10 *feet.*

The length of CD can be obtained either by measurement or by calculation. Thus, from the right-angled triangle ACD

$$AC^2 = AD^2 + DC^2.$$

As $AC = 10$ and $AD = 5$,

$$10^2 = 5^2 + DC^2;$$

$$\therefore\ DC^2 = 10^2 - 5^2 = 75,$$

$$DC = \sqrt{75} = 8{\cdot}66 \text{ ft.}$$

$$\text{Area of triangle} = \tfrac{1}{2}(10 \times 8{\cdot}66) = 43{\cdot}3 \text{ sq. ft.}$$

Using the second method

$$s = \tfrac{1}{2}(10 + 10 + 10) = 15.$$

Each remainder $= 15 - 10$ or 5.

$$\therefore\ \text{area of triangle} = \sqrt{15 \times 5^3}$$

$$= 43{\cdot}3 \text{ sq. ft.}$$

Area of a Hexagon.—If $ABCDEF$ (Fig. 65) is a regular hexagon, then six equilateral triangles are obtained by joining

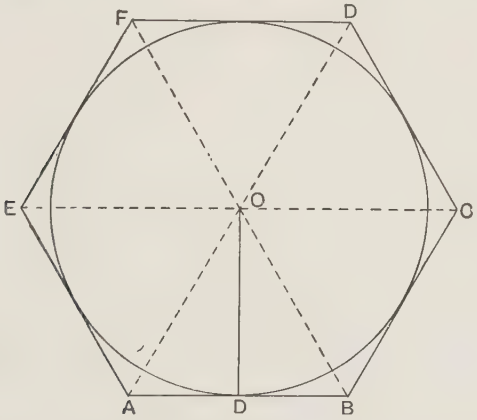


FIG. 65.—Area of a hexagon.

each angular point to the centre O. The area of one of these triangles can be found by the preceding method ; and six times the result will be the area required.

EXAMPLE 4. *Find the area of a regular hexagon each side of which is* 10 *ft.*

The area of an equilateral triangle each side of which is 10 ft. has been found to be 43·3 sq. ft. Hence, the area of the hexagon is $43{\cdot}3 \times 6 = 259{\cdot}8$ sq. ft.

The area of this or **any regular polygon** may be found by drawing the **inscribed circle**, then the area of the triangle AOB (Fig. 65) is $OD \times DA$, *i.e.* radius of circle multiplied by half the length of side AB.

∴ **Area of polygon = radius of inscribed circle multiplied by half the number of sides.**

EXERCISES XXXIII.

1. Find the area of a triangle, length of base 15 ft. and altitude 11 ft.

2. The area of a triangle is 13·13 sq. in. and the length of the base 7·5 in. Find its altitude.

3. The area of a triangle is 29·6 sq. ft. and the altitude 6·3 ft. Find the length of the base.

4. The area of the gable end of a house is 99·82 sq. ft. If the length of the base is 20 ft., what is the height from the base to the ridge?

5. Find the area of an equilateral triangle in which one side is 38 ft.

6. The length of the base of a triangular field is 24 chains, and the length of the perpendicular from the opposite angle is 14 chains. Find the area.

7. Find the area of a triangle, sides 13·6, 15 and 15·4 inches respectively.

What is the length of one of the equal sides of an isosceles triangle on a base of 14 in. having the same area?

8. Draw on squared paper a triangle whose sides are 9, 7 and 6 units and find its area. In what manner would you satisfy yourself of the truth of the formula $\sqrt{s(s-a)(s-b)(s-c)}$ for the area of the above triangle, where $s = \frac{1}{2}$(sum of sides) and a, b and c are the sides of the triangle?

9. Find the area of a triangle when the sides are 39 ft., 42 ft. and 45 ft. respectively.

10. What is the area (in square metres) of a triangle, base 45 cm. long and altitude 35 cm.?

11. The three sides of a triangular courtyard are 30, 40 and 50 ft. respectively. Find the cost of tiling it at 1*s.* per sq. ft.

12. Find (in acres) the area of a triangle, the sides of which measure 662, 442 and 808 links respectively.

13. Find the cost at £1 per acre of a triangular piece of ground the sides measuring 600, 700 and 800 yd. respectively.

14. The area of an equilateral triangle is 43·3 sq. ft. Find the length of a side.

15. The sides of a triangle are 10 ch., 8 ch., and 12 ch. respectively. What is the acreage of the field?

16. The perpendicular from the opposite angle on the base of a triangular field is 524 links, the length of the base is 25 chains. Find the area.

17. Find the area in sq. ft. of one end of a house consisting of a rectangle 30 ft. 9 in. wide and 20 ft. 3 in. high, surmounted by a triangle 10 ft. high.

Area of a Trapezium.—The area of a trapezium may be obtained by dividing the figure into two triangles. The sum of the areas of the two triangles will be the area required.

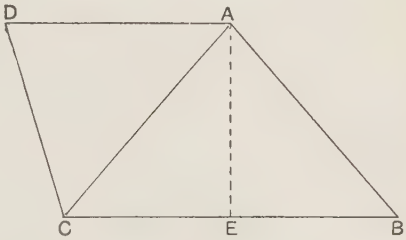


FIG. 66.—Area of a trapezium.

Let $ABCD$ be a given trapezium; then, if a line AC be drawn (Fig. 66) the figure is divided into two triangles, ABC and ADC. Draw AE perpendicular to BC meeting BC in E. Then

$$\text{area of triangle } ABC = \tfrac{1}{2}(BC \times AE),$$

$$\text{,, ,, } ADC = \tfrac{1}{2}(AD \times AE);$$

$$\therefore \text{ area of trapezium} = \tfrac{1}{2}AE(BC + AD).$$

Or, **the area of a trapezium is one-half the sum of the parallel sides multiplied by the perpendicular distance between them.**

EXAMPLE 1. *The parallel sides of a trapezium are 12 ft. and 8 ft. respectively, and the perpendicular distance between them is 7 ft. Find the area.*

Here AE (Fig. 66) = 7 ft., BC and AD are 12 and 8 ft. respectively.

$$\text{Area of figure} = \tfrac{1}{2}(12+8)7 = 70 \text{ sq. ft.}$$

EXAMPLE 2. *The area of a field in the form of a trapezium is 3·1 acres. If the two parallel sides are 700 and 300 links respectively, what is the perpendicular distance between them?*

$$1 \text{ ac.} = 100000 \text{ sq. links},$$

$$3{\cdot}1 \text{ ac.} = 3{\cdot}1 \times 100000 = 310000 \text{ sq. links}.$$

Half sum of parallel sides $= \tfrac{1}{2}(700+300) = 500$;

$\therefore$ perpendicular distance $= \frac{310000}{500}$

$= 620$ links.

Area of a Quadrilateral.—The area of any quadrilateral figure can be obtained by dividing the figure into two triangles.

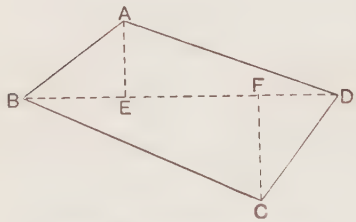


FIG. 67.—Area of a quadrilateral.

Let $ABCD$, Fig. 67, be a given quadrilateral. Draw a diagonal DB; and draw AE and CF perpendicular to DB. Then the area of the four-sided figure $ABCD$ is equal to the area of the triangle ADB together with the area of DBC.

$$\text{Area of triangle } ADB = \tfrac{1}{2}(DB \times AE);$$

$$\text{,, ,, } DCB = \tfrac{1}{2}(DB \times CF);$$

$$\therefore \text{ area of quadrilateral} = \tfrac{1}{2}DB(AE+CF).$$

Or, **the area of a quadrilateral is one-half the product of one of the diagonals of the figure and the sum of the perpendiculars let fall on that diagonal from the two opposite angular points.**

EXAMPLE 1. *The diagonal of a quadrilateral is* 300 *yd., and the perpendiculars let fall from the opposite angles are* 10 *yd. and* 15 *yd. respectively. Find the area.*

$$\text{Area} = \tfrac{1}{2} \times 300(10+15) = 150 \times 25$$
$$= 3750 \text{ sq. yd.}$$

EXERCISES XXXIV.

1. The area of a trapezium is 15548 sq. ft., the parallel sides are 234 ft. and 104 ft. respectively. Find the perpendicular distance between the parallel sides.

2. The plan of a district is a trapezium having its parallel boundaries 276 and 216 miles respectively. Find the area when the perpendicular distance between the parallel sides is 72·5 miles.

3. The area of a field in the form of a trapezium is 6 acres 3 roods. The parallel sides are 10 chains 30 links and 7 chains 70 links respectively. Show that the perpendicular distance between the parallel sides is 7 chains 50 links.

4. A trapezium has two of its parallel sides 10 and 12 ft. respectively. Find the perpendicular distance between the parallel sides if the area of the figure is 44 sq. ft.

5. The two parallel sides of a trapezium are 27·5 ft. and 38·5 ft. The perpendicular distance between them is 12·38 ft. Find the area.

6. One of the parallel sides of a trapezium is 4 ft. longer than the other and the perpendicular distance between them is 7 ft. If the area is 70 sq. ft., find the lengths of the parallel sides.

7. A section of a railway cutting in the form of a trapezium is 14 ft. wide at the bottom, and 26 ft. wide at the top. What is the area of a cross-section if the depth is 20 ft.?

8. A piece of land in the form of a quadrilateral $ABCD$ has its diagonal DB - 150 yards. The perpendiculars from the angles A and B on the diagonal are 24 ft. and 30 ft. respectively. Find the area.

9. The area of a quadrilateral is 6250 sq. yd. The perpendiculars from the opposite angles on the diagonal are 20 and 30 yd. respectively. Find the length of the diagonal.

10. The area of a quadrilateral is 10500 sq. ft., the diagonal being 300 ft. If the length of the perpendicular from one of the angles on the diagonal is 49 ft., what is the length of the other diagonal?

CHAPTER VII.

Circumference of Circle. Circumference of Ellipse. Arches. Area of Circle, Ellipse. Area of Irregular Figure. Simpson's Rule. Slide Rule. Surface of Prism, Cylinder, Pyramid, Cone, Sphere.

Circumference of a Circle.—The number of times that the circumference of a circle contains the length of the diameter cannot be exactly expressed as a decimal fraction, but it is very nearly equal to 3·1416, a value which is usually denoted by the Greek letter π (pi). Either of the numbers 3·142 or $3\frac{1}{7}$ are sufficiently exact for nearly all practical purposes; the former value is generally used with four-figure logarithms.

The numerical value of π may be determined by experiment in many ways:

(i) It may be found by means of a tape measure, or a piece of thin paper, wrapped round a circular disc or cylinder. In this way the circumference is obtained. The diameter is now measured and then simple division gives the ratio required.

(ii) Roll a disc or cylinder of wood or metal on a scale. Make a mark on the circumference of the cylinder, and put the mark coincident with a particular scale division. Roll the cylinder along until the mark is again coincident with a scale division, taking precautions to prevent slipping. The distance in scale divisions moved through by the cylinder in one revolution will be found to be $3\frac{1}{7}$ times the diameter.

(iii) By drawing a semicircle of any convenient size, then applying a strip of tracing paper on which a straight line has been drawn (as on p. 142), a line equal in length to the

semicircle may be obtained. This length is one-half the circumference required.

EXAMPLE 1. *The diameter of a circle is* 18·5 *inches; find its circumference.*

$$\text{Circumference} = 18{\cdot}5 \times 3{\cdot}142 = 58{\cdot}13 \text{ in.}$$

The product may be found either by contracted multiplication or by logarithms.

3·142	
581	$\log 18{\cdot}5 = 1{\cdot}2672$
3142	$\log 3{\cdot}142 = 0{\cdot}4972$
2514	$1{\cdot}7964$
157	
58·13	$\text{antilog } 1{\cdot}7644 = 58{\cdot}13.$

EXAMPLE 2. *The circumference of a circle is* 21·99 *in.; find the diameter.*

$$\text{Diameter} = 21{\cdot}99 \div 3{\cdot}142 = 6{\cdot}998 \text{ in.}$$

$\log 21{\cdot}99 = 1{\cdot}3422$
$\log 3{\cdot}142 = 0{\cdot}4972$ $\qquad \text{antilog } 0{\cdot}8450 = 6{\cdot}998.$
$0{\cdot}8450$

EXAMPLE 3. *How many turns will a wheel* 40·33 *in. diameter make in* 6 *miles?*

In one turn the wheel moves a distance in feet equal to

$$40{\cdot}33 \times 3{\cdot}142 \div 12.$$

$$6 \text{ miles} = 6 \times 5280 \text{ ft.}$$

$$\text{Number of turns} = \frac{6 \times 5280 \times 12}{40{\cdot}33 \times 3{\cdot}142} = 3000.$$

$\log 6 = 0{\cdot}7782$	
$\log 5280 = 3{\cdot}7226$	$\log 40{\cdot}33 = 1{\cdot}6056$
$\log 12 = 1{\cdot}0792$	$\log 3{\cdot}142 = 0{\cdot}4972$
$5{\cdot}5800$	$2{\cdot}1028$
$2{\cdot}1028$	
$3{\cdot}4772$	$\text{antilog } 3{\cdot}4772 = 3000.$

To find the Length of a Curved Line.—The length of the curved line *ADB* (Fig. 68) may be determined

approximately by using a strip of *tracing paper* on which a straight line has been drawn. One end of this line can be made to coincide with A (Fig. 68) and a short portion of the line to coincide with the curve; then by using a sharp point, such as the point of a needle, by moving the line about such a point it may be made to coincide with another small length of the curve. By changing to P, another portion is added, and so on until the whole length of the curve has been transferred to the line on the tracing paper. The length thus obtained can be ascertained by using a scale, or by placing the line on squared paper and so noting its length.

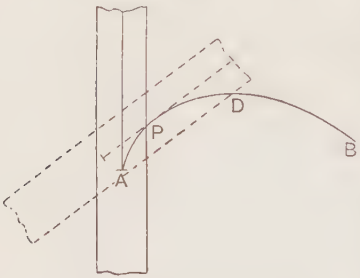


FIG. 68.—Length of a curved line.

In the case of a circular arc the length may also be obtained by calculation. Thus, bisect AB at F and at F draw FD, perpendicular to AB, meeting the circular arc at D. Join AD, then AB is called the **chord of the arc** and AD the **chord of half the arc.**

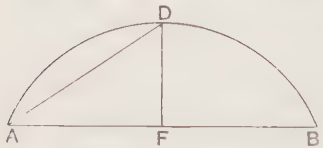


FIG. 69.—Chord of an arc and chord of half an arc.

When the lengths of AB and AD are known, the length of the arc ADB may be obtained approximately by the following formula:

$$L = \text{length of arc} = \frac{8 \times AD - AB}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Any two of the three terms in (i) being given, the remaining term may be found.

In the case of an arch the distance AB is called the **span** of the arch and FD is the **rise** or **height** of the arch.

If the span and the rise of an arch are given, then as AFD (Fig. 70) is a right-angled triangle, the hypotenuse AD is the square root of the sum of the squares of the remaining sides AF and FD.

$$\therefore\ AD=\sqrt{(AF)^2+(FD)^2}.$$

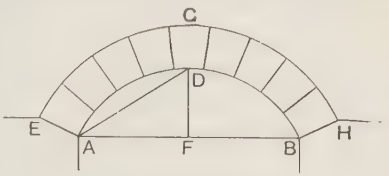


Fig. 70.—Arch.

Having obtained the inner curved line ADB called the **intrados** or **soffit** of the arch, the outer curve EGH, called the **extrados** or **back** of the arch can be obtained.

Given the height and the span, to find an arc of a circle passing through the three points.—Mark off on squared paper the given height FD and span AB; then the arc of a circle passing through the three points A, D and B, may be obtained as follows:

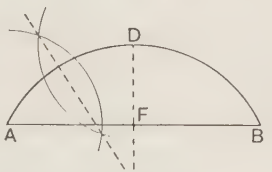


Fig. 71.—Circle through three points.

The centre of the arc will lie at some point in a line FD bisecting AB and at right angles to it. To obtain the centre, describe arcs with A and D as centres and any convenient length as radius as shown (Fig. 71). Join the points of intersection of these arcs, and produce the line if necessary to intersect DF in O. With O as centre and radius OA, or OB, describe an arc of a circle through the three given points ADB.

Example 1. *The circular arch of a bridge is 55 ft. long and the chord of half the arc is 26·9 ft. Find the length of the chord or span.*

Let l denote the length; then substituting the given values in Eq. (i) p. 142, we find

$$55=\frac{8\times 26{\cdot}9-l}{3},$$

$$\text{or}\quad l=8\times 26{\cdot}9-3\times 55=50{\cdot}2\text{ ft.}$$

EXAMPLE 2. *The chord of an arc of a circle is 136 ft. and the chord of half the arc is 75·5 ft. Find the length of the arc.*

$$\text{Length of arc} = \frac{8 \times 75{\cdot}5 - 136}{3}$$
$$= 156 \text{ ft.}$$

EXAMPLE 3. *The span of an arch is to be 30 ft. : the length of the arch or soffit 56 ft. 8 in. Find the length of the half chord and the rise or height of the arch.*

$$56 \text{ ft. } 8 \text{ in.} = 56\tfrac{2}{3} \text{ ft.}$$

If l denotes the length of the half chord, then

$$56\tfrac{2}{3} = \frac{8l - 30}{3},$$
$$\text{or} \quad 8l = 170 + 30 = 200;$$
$$\therefore \ l = \tfrac{200}{8} = 25 \text{ ft.}$$
$$h = \sqrt{25^2 - 15^2} = \sqrt{(25+15)(25-15)}$$
$$= \sqrt{400} = 20 \text{ ft.}$$

Ellipse.—An ellipse may be obtained by projecting a circle on a plane to which it is inclined. Or, if a circle and its vertical and horizontal diameters be drawn on a sheet of indiarubber and the sheet be stretched uniformly, say by means of two strips of wood, the circle becomes an ellipse. The horizontal diameter is lengthened and the vertical diameter shortened. The longer diameter or *axis* AA' (Fig. 72) is called the **major axis** and the shorter axis BB' the **minor axis** of the ellipse.

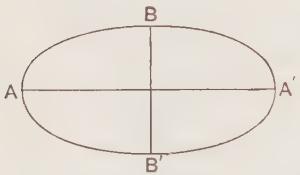


FIG. 72.—Ellipse.

Several methods may be used to draw the curve when the lengths of the axes are given; it is only necessary to refer to one of the most important.

Draw a line AA' (Fig. 73) equal in length to the major axis, and the minor axis BB' perpendicular to and bisecting AA'.

Mark on a strip of paper, or thin cardboard, a length DF equal to half the major axis, and a length EF equal to half the minor axis; place the strip so that the points E and

F coincide with the two perpendicular lines intersecting as in Fig. 73, then D is a point on the required curve. When the strip is moved into another position, and the points E and F made to coincide with the lines AA', BB', then another point on the curve is obtained. In this manner any number of points on the curve may be found and finally a fair curve drawn through the points so determined will give the ellipse required.

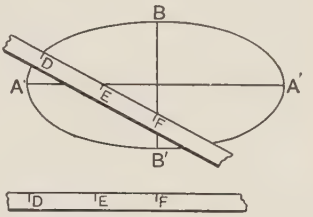


FIG. 73.—Construction of an ellipse.

Circumference of an Ellipse.—The circumference or perimeter of an ellipse (except in those cases when the major axis is many times the minor axis) may be obtained to a fair degree of approximation from the formula

$$\text{circumference} = \pi(a+b),$$

where a denotes the semi-major axis $\frac{AA'}{2}$,

and b denotes the semi-minor axis $\frac{BB'}{2}$.

EXAMPLE 1. *The major axis of an ellipse is 5 in. long and the minor axis 4 in. Verify that the circumference is* $\pi(2{\cdot}5+2) = 4{\cdot}5\pi = 14{\cdot}14$ *inches.*

Draw an ellipse of the dimensions given by the method already described. Draw on a strip of tracing paper a straight line and put one end of the line coincident with A (Fig. 74) and a short length of the line to coincide with the curve; insert a needle point at the place where the line leaves the curve and rotate the line about this point as centre until another short length of the line coincides with the curve. Repeat the process until a line equal in length to the perimeter of the ellipse has been obtained. Measure this line and compare the result with that obtained by using the formula $\pi(2{\cdot}5+2) = 4{\cdot}5\pi = 14{\cdot}14$ in.

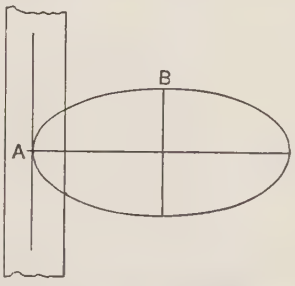


FIG. 74.—Circumference of an ellipse.

EXAMPLE 2. *Draw an ellipse in which the major and minor axes are 5 in. and 1 in. long respectively. Obtain the perimeter by using a strip of tracing paper and by calculation. Compare the results.*

$$\text{Perimeter} = \pi\left(\tfrac{5}{2}+\tfrac{1}{2}\right) = 9{\cdot}4 \text{ in.}$$

EXAMPLE 3. *Find the perimeter of an ellipse in which the axes are 50 and 40 cm. respectively.*

Here $a = 25$, $b = 20$;

$$\therefore \text{ perimeter} = \pi(25+20) = 45\pi$$
$$= 141{\cdot}4 \text{ cm.}$$

$\log 45 = 1{\cdot}6532$
$\log \pi = 0{\cdot}4972$
$2{\cdot}1504$ $\qquad$ antilog $2{\cdot}1504 = 141{\cdot}4$.

EXAMPLE 4. *The major axis of an ellipse is 42 in. long and the perimeter 119·4 in. Find the length of the minor axis.*

Let b denote the length of the semi-minor axis;

$$\therefore \quad 119{\cdot}4 = \pi(21+b);$$
$$\therefore \quad 119{\cdot}4 = 21\pi + b\pi,$$
$$\text{or} \quad b = \frac{119{\cdot}4 - 21\pi}{\pi} = \frac{53{\cdot}42}{\pi}$$
$$= 17;$$
$$\text{minor axis} = 34 \text{ in.}$$

EXERCISES XXXV.

[$\pi = 3{\cdot}142$.]

1. Find the circumference of a circle when the diameter in inches is (i) 8·3, (ii) 9·7, (iii) 54·6, (iv) 76·4.

2. Find the diameter of a circle when the circumference in inches is (i) 223, (ii) 97·7, (iii) 103, (iv) 133·5.

3. A wheel 5 ft. diameter made 2137 revolutions in 6 miles. Find what distance is due to slipping of the wheels.

4. How many turns per minute will be made by a carriage wheel 4·55 ft. diameter, at the rate of 10·4 miles in 1·25 hour?

5. If the mean diameter of the earth be 7912 miles, what is the circumference (i) assuming $\pi = 3{\cdot}142$, (ii) $\pi = 3{\cdot}1416$.

6. The diameter of a pond is 63·66 ft. What will it cost to enclose it with a fence at 6*d.* per foot length?

7. A wheel has 200 teeth, 2 inches pitch. Find the diameter of the pitch circle.

[The *pitch* is the distance between the centres of two consecutive teeth measured on the circumference of the pitch circle.]

8. If the circumference of a cart wheel is 12 ft. 6 in., what is its diameter? How often will it revolve in 8 miles?

9. The major and minor axes of an ellipse are 5 in. and 4 in. respectively. Find the diameter of a circle the perimeter of which is equal to that of the ellipse.

10. An elliptical cast-iron manhole door 3 ft. by 2 ft. is to have a wrought-iron hoop round its periphery. Find the length of the hoop, allowing 2 in. for overlap.

Area of a Circle.—The area of a circle may be found by multiplying the square of the diameter by 0·7854; or if d denotes the diameter, then the area is $0{\cdot}7854d^2 = \pi r^2$, where r is the radius of the circle.

The formula may be verified in various ways:

(i) If the circumference of a thin disc be divided into a large number of equal parts as in Fig. 75, and all these

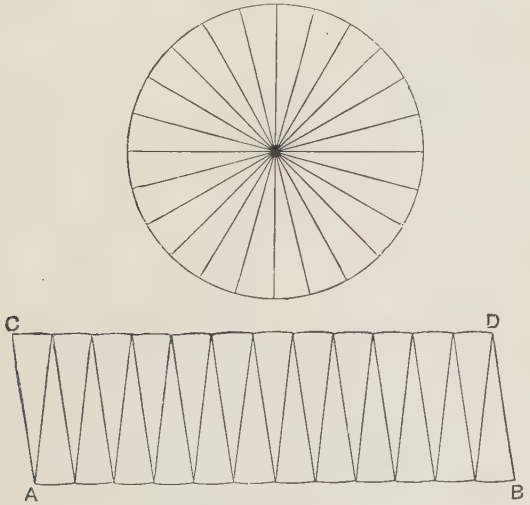


Fig. 75.—Area of a circle.

points are joined to the centre, a number of equal sectors may be obtained. If these sectors are cut out (say by a fine saw),

then, by means of a piece of tape glued to the half circumference AB, the sectors may be arranged on a line as in Fig. 75. The sectors on the upper half may be fitted into the corresponding recesses in the lower half. In this manner a parallelogram is obtained, the base being $3{\cdot}1416 \times r$ and the height r. Hence, the area of the circle equals $3{\cdot}1416r^2$ or πr^2.

As $r = \frac{d}{2}$, $r^2 = \frac{d^2}{4}$, also $\frac{3{\cdot}1416}{4} = 0{\cdot}7854$;

$$\therefore \pi r^2 = 0{\cdot}7854d^2.$$

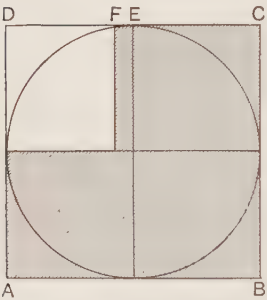


Fig. 76.—Area of a circle.

(ii) Two circles may be drawn on the same sheet of cardboard, and a square drawn touching the circumference of one of the circles as in Fig. 76. The horizontal and vertical diameters divide the square into four equal squares; the side of each square is equal to r, and the area of each small square is r^2. Make $EF = \frac{1}{7}ED$; then the area of the shaded figure is obviously $3\frac{1}{7} \times r^2$. Cut the circle out carefully, and also the shaded part of the square. If this is done, the weights of each will be found to be the same. The areas are proportional to the weights, and as the thickness of one is equal to the thickness of the other and the weights are equal, the areas also are equal.

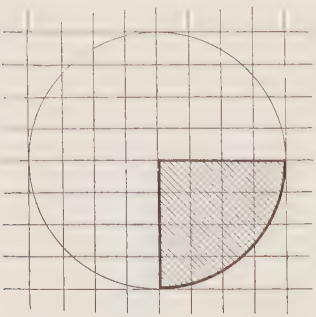


Fig. 77.

(iii) The circle may be drawn on squared paper and the number of squares enclosed obtained. This process is easily carried out by counting the enclosed squares in a quadrant and multiplying the result by 4 (Fig. 77). As the curved line of the quadrant will cut some

of the squares, the rule adopted is to reckon as full squares all those squares cut by the circular arc which are greater than half a square, and to neglect those which are less.

Hollow Circle or Annulus.—When, as in Fig. 78, two circles of unequal size are described from the same centre, the portion between the two circles is usually called an **annulus.** The area is found obviously by subtracting the area of the inner circle from that of the outer circle.

Using D to denote the diameter of the outer, and d the diameter of the inner circle, the area of the annulus is

$$0{\cdot}7854D^2 - 0{\cdot}7854d^2$$
$$= 0{\cdot}7854(D^2 - d^2). \quad \ldots\ldots(i)$$

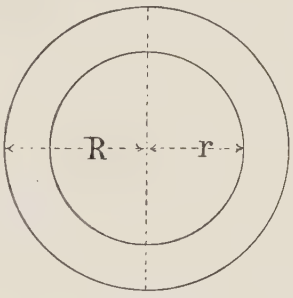


FIG. 78.—Area of an annulus.

As the difference of the squares of two numbers is equal to the product of the sum and difference of the numbers (p. 131), the preceding equation may be written as

$$\text{area} = 0{\cdot}7854(D+d)(D-d),$$
$$\text{or, area} = \pi(R^2 - r^2) = \pi(R+r)(R-r),$$

where R denotes the radius of the outer and r the radius of the inner circle.

EXAMPLE 1. *The external diameter of a steel shaft is* 20 *in. and the internal diameter* 16 *in.; find the area of the cross-section.*

$$\text{Area} = 0{\cdot}7854(20^2 - 16^2) = 0{\cdot}7854(20+16)(20-16)$$
$$= 0{\cdot}7854 \times 36 \times 4 = 113{\cdot}1 \text{ sq. in.}$$

$$\log 0{\cdot}7854 = \bar{1}{\cdot}8951$$
$$\log 36 = 1{\cdot}5563$$
$$\log 4 = 0{\cdot}6021$$
$$2{\cdot}0535 \qquad \text{antilog } 2{\cdot}0535 = 113{\cdot}1.$$

Area of an Ellipse.—The area of an ellipse may be ascertained by drawing the figure on squared paper and

counting the enclosed squares. Or, when drawn upon cardboard it may be cut out and weighed: then, from a rectangle or square cut from the same sheet, the weight of a square inch can be ascertained and hence the area of the ellipse.

The area may also be obtained from the formula:

$$\text{area} = \pi ab.$$

This equation may be verified as follows:

Draw on the same sheet of cardboard two ellipses of the same size, *i.e.* major axis $2a$ and minor axis $2b$ (Fig. 79). Draw a rectangle circumscribing one ellipse, then if BE be made equal to $\frac{1}{7}BF$ the area of the shaded figure will be $3\frac{1}{7}(a \times b)$. If this shaded figure be placed in one pan of a balance and the remaining ellipse in the other, then the two will be found to balance, hence, the area of the ellipse is $\pi \times ab$.

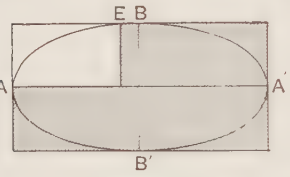


Fig. 79.—Area of an ellipse.

Example 1. *Find the area of an ellipse in which the axes are* 50 and 40 *cm. respectively.*

Here $a = 25$, $b = 20$;

$$\therefore \text{ area} = \pi(25 \times 20) = 500\pi$$
$$= 1571 \text{ sq. cm.}$$

$\log 500 = 2{\cdot}6990$
$\log \pi = 0{\cdot}4972$
$3{\cdot}1962$ antilog $3{\cdot}1962 = 1571$.

Example 2. *The minor axis of an ellipse is* 8 *cm. long. What is the length of the major axis if the area of the ellipse is* 200 *sq. cm.?*

Let a denote the length of the semi-major axis.

Then $$\text{area} = \pi(a \times 8) = 200;$$

$$\therefore a = \frac{200}{8\pi} = \frac{25}{\pi} = 7{\cdot}956;$$

$$\therefore \text{major axis} = 2 \times 7{\cdot}956 = 15{\cdot}91.$$

$\log 25 = 1{\cdot}3979$
$\log \pi = 0{\cdot}4972$
$0{\cdot}9007$ antilog $0{\cdot}9007 = 7{\cdot}956$.

EXERCISES XXXVI.

1. Find the area of a circle, diameter 2·47 inches.

2. Find the area of a circle, diameter 3·25 inches.

3. The diameters of three pieces of brass wire in inches are as follows: (i) 0·1048, (ii) 0·0165, (iii) 0·025. Find in each case the area of the cross-section.

4. Find the diameters of three pieces of wire whose areas of cross-section are respectively twice those in Exercise 3.

5. The diameters of three lengths of steel wire are in inches: (i) 0·256, (ii) 0·0343, (iii) 0·072. Find in each case the area of cross-section.

6. Find the diameters of three lengths of wire whose sectional areas are respectively one-half of those in Exercise 5.

7. A circular bicycle track has an internal radius of 150 yd. and is 5 yd. wide. Find the area of the track. What distance does a rider who keeps to the inside of the track save over one who keeps on the outside?

8. Find the area of a cross-section of a cast-iron pipe, external diameter 12 in., internal diameter 11 in.

9. What is the area of cross-section of a cylinder, external diameter 10 in. and thickness of metal $\frac{3}{8}$ in.?

10. The external diameter of a hollow steel shaft is 14 in. and internal diameter 10 in. Find the area of a cross-section.

11. A hole 4 in. diameter is bored through a cast-iron coupling 8 in. diameter. What is the area of a cross-section?

12. Draw on coloured paper a circle 3 in. diameter. Divide the circumference into 12 equal parts and join the points to the centre, thus obtaining 12 equal sectors. Cut these out and paste on a sheet of white paper so as to form a parallelogram. Find the area.

13. The circumference of a circle is 2 ft. Find its area.

14. The diameter of a piston is 2 ft. 6 in. Find its area.

15. Find the difference in pressure on the two sides of a piston, if the diameter of the piston is 2 ft. 6 in. and the diameter of the piston rod on one side is 5 in., pressure of steam 100 lb. per sq. in.

16. A circular steel plate 3 ft. 8 in. diameter is subjected to a pressure of 15 lb. per sq. in. Find the total pressure on the plate in tons.

17. A circular brick tower has an outer circumference of 18·34 metres and an inner circumference of 14·74 metres. Find the area covered by the wall.

18. What is the area in square feet of an elliptical manhole door, longer diameter 19 in., shorter diameter 17 in.?

19. The longer diameter of an elliptical plate is 29¼ in., and the shorter diameter 27½ in. Find its periphery and area.

Area of an Irregular Figure.—None of the preceding methods is applicable when one or more sides of a plane figure are formed by curves. Thus, to obtain the area of the figure *EFGH* (Fig. 80), draw a line *EF* on squared paper and divide it into any convenient number of equal parts 6, 8, 10, etc. At each point of division erect perpendiculars; these are called *ordinates*. When the length of the base *EF* and the ordinates are given, a series of points may be obtained through which a fair curve may be drawn, and the area found either by counting the squares enclosed or by calculation, using rules such as the **mid-ordinate** or **Simpson's rule**.

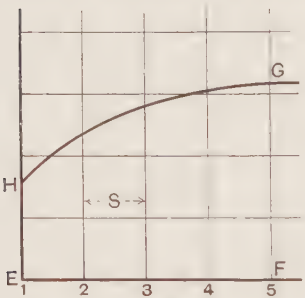


Fig. 80.

Mid-ordinate Rule.—The base *EF* is divided into any convenient number of parts, and perpendiculars are drawn at the centre of each as shown by dotted lines in Fig. 81. The lengths of all these lines are added together and the sum of these lengths divided by the number of ordinates. The result gives the **mean** or **average ordinate**, and the product of this value and the length of the base is the area required. A more accurate method is that known as **Simpson's rule**; the

proof cannot be given here, but may be found in the author's *Manual of Practical Mathematics* (Macmillan), p. 417.

Simpson's Rule.—Divide the base EF (Fig. 80) into an *even* number of equal parts, thus obtaining an *odd* number of ordinates. Starting at E, number the ordinates 1, 2, 3, 4, 5, 6, 7, ... The ordinates at E and F are called the *end ordinates*, the ordinates 2, 4, 6, ..., are called the *even* ordinates and 3, 5, 7, ..., the *odd* ordinates. Find the sum of the end ordinates, next the sum of the even ordinates; now multiply the sum of the even ordinates by 4: find the sum of the odd ordinates 3, 5, 7, ..., and multiply their sum by 2; add all these sums together and multiply by one-third the common distance between the ordinates.

This calculation may be expressed in a simpler manner as follows:

Let A denote the sum of the end ordinates

B „ „ „ even „

C „ „ „ odd „

Then the area of the figure is given by

$$\frac{s}{3}(A+4B+2C),$$

where s is the common distance between the ordinates.

If the given figure is symmetrical about a line such as EF, the area $EFGH$ will be half the area required, and this multiplied by 2 will be the area of the figure.

Example 1. *The half-ordinates of a transverse section of a vessel are in feet*, 12·2, 12·2, 12·1, 11·8, 11·2, 10·0 *and* 7·3 *respectively. The common distance apart is* 18 *inches. Find the area of the section in square feet* (i) *by the mid-ordinate*, (ii) *by Simpson's rule.*

As the lowest numerical value of an ordinate is 7·3 the vertical scale may commence as in Fig. 81 at 6. By taking 1 division to represent 1 ft., points on the curve are obtained and a fair curve can be drawn through them. The mid-ordinates shown in dotted lines may be drawn, and when read off are found to be 12·2, 12·18, 12·0, 11·6, 10·7

and 8·95 ft. respectively. The sum is 67·63 and as there are six ordinates the average ordinate is

$$\frac{67\cdot63}{6}.$$

Hence, area of figure $= \frac{67\cdot63}{6} \times 9.$

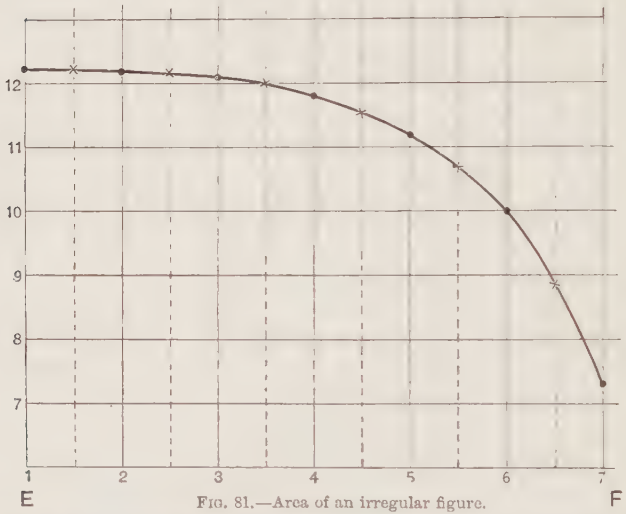


Fig. 81.—Area of an irregular figure.

Also, as the figure is symmetrical about the line EF, the area is

$$\frac{67\cdot63 \times 18}{6} = 202\cdot89 \text{ sq. ft.}$$

Using Simpson's rule.

Sum of end ordinates $= 12\cdot2 + 7\cdot3 = 19\cdot5$

,, even ,, $= 12\cdot2 + 11\cdot8 + 11 = 34$

,, odd ,, $= 12\cdot1 + 11\cdot2 = 23\cdot3.$

Common distance is 18 in. $= \frac{3}{2}$ ft.

Half area $= \frac{1}{3} \times \frac{3}{2}(19\cdot5 + 4 \times 34 + 2 \times 23\cdot3) = 101\cdot05$;

$\therefore$ area $= 202\cdot1$ sq. ft.

The end ordinates may be zero, the curved portion starting and finishing on the line EF. Also, when the ordinates are given, it is

necessary in using the *mid-ordinate* rule to draw the curve and obtain the lengths of the mid-ordinates. This procedure is not necessary in Simpson's rule.

EXAMPLE 2. *The ordinates of a curve are* 0, 0·72, 1·15, 1·33, 1·22, 0·81 *and* 0 *inches. If the common distance between the ordinates is* 1 *inch, find the area* :

(i) *Using the mid-ordinate rule ;*
(ii) *Using Simpson's rule.*

(i) The given values may be set off on squared paper as in Fig. 82 and the curve passing through the points obtained. The mid-ordinates are

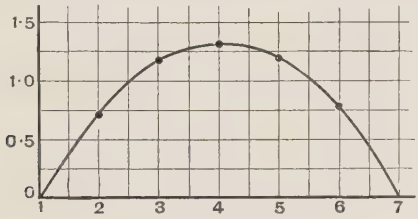


FIG. 82.—Area of an irregular figure.

read off and are found to be 0·37, 0·95, 1·25, 1·3, 1·05, 0·4 inches. The sum is 5·32 ;

$$\therefore \text{ area} = \frac{5{\cdot}32 \times 6}{6} = 5{\cdot}32 \text{ sq. in.}$$

(ii) Sum of end ordinates = 0,
,, even ,, = 0·72 + 1·33 + 0·81 = 2·86,
,, odd ,, = 1·15 + 1·22 = 2·37.

$$\text{Area} = \tfrac{1}{3}(0 + 2{\cdot}86 \times 4 + 2{\cdot}37 \times 2) = 5{\cdot}39 \text{ sq. in.}$$

EXERCISES XXXVII.

1. The semi-ordinates of the midship section of a vessel are 0·2, 3·6, 7·4, 10, 11, 10·7, 9·3, 6·3, 2 ft. respectively. What is the area of the section when the ordinates are 15 ft. apart?

2. The half-ordinates of a midship section are 14·4, 33·6, 49·6, 57·6, 60, 57·6, 50, 40 and 25·6. The common distance between the ordinates is 30 ft. Find the area of the section.

3. The half-ordinates of the midship section of a vessel are 12·7, 12·8, 12·9, 12·9, 12·9, 12·8, 12·5, 11·9, 10·4, 5·9 and 1·4 ft. respectively, the common distance 18 in. Find the area of the section.

4. Find the area of half of a ship's midship section of which the curved form is defined by the following equidistant ordinates spaced 12 ft. apart: 0·1, 5·1, 7·17, 8·75, 10·1, 9·17, 8·05, 6·4, 0·1 ft.

5. Find the area of a vessel's whole section of which the half ordinates are as follows: 0·2, 3·7, 4·8, 5·1, 4·8, 4·0, 0·2, the interval between them being 9 ft.

6. The half ordinates of the deck plan of a vessel are 0·5, 7, 12, 14·8, 15·6, 16, 16, 15·6, 14·6, 11·2 and 3·3 feet respectively. Find the area of the deck plan in square yards, length of deck plan 158 ft.

7. The ordinates of a curved piece of sheet lead in inches are as follows: 30, 30, 29·9, 29·5, 28·4, 25·7, 14·2, the common distance between them being 3·65 inches. Find the area of the sheet.

8. The ordinates of a curved sheet of steel plate are 0·1, 0·5, 11·6, 15·4, 16·8, 17, 16·9, 16·4, 14·5, 9·4 and 0·1 inches, the common distance being 11 inches. Find the weight of the plate if the weight per square foot is 20 lb.

9. Draw a semicircle 2·6 in. radius and use Simpson's rule to obtain its area. Find the percentage difference of the result from that obtained by using $3{\cdot}142 \times 2{\cdot}6^2 \div 2$.

10. Draw a quadrant of a circle, radius 2 in., and draw the tangents at its extremities. Use Simpson's rule to find the area included between the circumference and the tangents. Also find the percentage error.

Slide Rule.—The processes of multiplication and division are converted by logarithms into mere addition and subtraction, hence, if the logarithms of numbers are marked on a suitable scale, the processes of addition and subtraction corresponding to multiplication and division may be carried out readily.

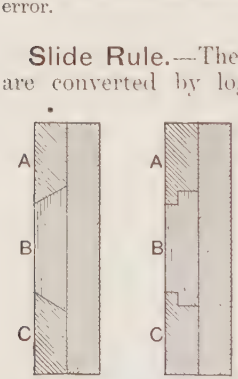


FIG. 83.—Sections of slide rule.

Perhaps one of the best methods is to make a slide rule. Even if the result is only a crude representation it will probably be sufficient to give a clear insight into the manipulation of the instrument.

To make the rule, four slips of thin wood are used. One piece forms the back; two narrower pieces with edges bevelled, or recessed, as in Fig. 83, form

the guides A and C; and the fourth piece forms a slider which is made to move easily between these guides, but is prevented from falling away by the bevelled or recessed surfaces.

Strips of white paper should be pasted to the wooden strips to enable the scales to be marked on them. Cardboard may be used, of course, to replace the wooden strips.

Use a scale divided into tenths of an inch, and on the guide A measure a distance 0·301 in. from 1 to 2. Mark this division as 2. From 1 to 3 measure a distance 0·477 in. and mark it as 3. Or, since these distances are comparatively small, it may be advisable to multiply each length by 10 and to proceed to mark off the distances 3·01 and 4·77 in. The sum of the distances from 1 will thus be

$$3{\cdot}01 + 4{\cdot}77 = 7{\cdot}78 \text{ in.}$$

This division is marked 6. The three divisions 1, 2 and 3 are copied on to the slider B.

Now put 1 on the slider coincident with 2 on the scale; opposite 3 on the slider 6 will be found on the scale, or $2 \times 3 = 6$.

The converse process is easily seen; thus, put 3 on the slider coincident with 6 on the scale; then opposite 1 on the slider will be found 2 on the scale; or $6 \div 3 = 2$.

The scale may now be completed by measuring distances 6·021, 6·99, 8·45, 9·03 and 9·54 from 1 and marking these 4, 5, 7, 8 and 9 respectively. Finally these divisions are copied on the slider.

The logarithms of the fractional numbers 1·5, 2·5, etc., may be marked on the scale and slider and the processes of multiplication and division will be easily made out.

These two scales on a slide rule (Fig. 84) are usually known as the **A and B scales**; but if the scales on an actual slide rule are examined, two more scales will be found, known as the **C and D scales**. The two scales C and D are exactly alike, the former being marked on the slider and the latter on the guide D. The lower divisions, however, are one-half the upper. Thus, the distance 6·02, when measured along the lower scale, is not marked 4 as in the upper scale, but 2. Similarly, under 9, 3 is found.

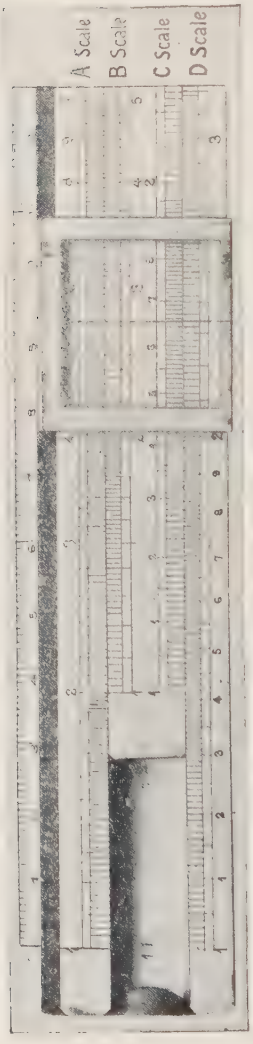


FIG. 84.—End of a slide rule.

The square root of any number on the upper scale is found immediately below it on the lower scale. Conversely, the square of any number on the lower scale is immediately above it on the upper scale. To enable the divisions on the lower and upper scales to be read off easily, a **cursor** is provided and the coincidence of any two divisions is obtained readily. The cursor is also useful when the product or quotient of several factors are required. Thus, the cursor may be set to the product or quotient of two factors, and the multiplication or division by another factor obtained without reading off the preceding result.

EXAMPLE 1. *Find the area of a circle 3 inches diameter.*

The area may be found either from $0{\cdot}7854 \times 3^2$ or $3{\cdot}142 \times 1{\cdot}5^2$.

Opposite 3 on the lower scale of the slide rule its square 9 is found on the upper scale. Put 1 on the slider coincident with 9 on the upper scale, 0·785 on the slider is found coincident with 7·01 on upper scale.

Or, using $\pi \times 1{\cdot}5^2$, the square of 1·5 is 2·25 and this multiplied by π (marked on the slider) gives 7·01 as before.

Hence, the area is 7·01 sq. in.

EXAMPLE 2. *The diameter of a circle is 6·5 in. Find its circumference.*

Put 1 on the slider coincident with 6·5 on the *A* scale, then opposite the mark (π) on the slider is the answer 20·4 on the *A* scale.

EXAMPLE 3. *The circumference of a circle is* 10·2 *in. Find its diameter.*

Put the mark π on the slider coincident with 10·2 on the A scale, then opposite 1 on the slider is the answer 3·25 on the A scale.

EXAMPLE 4. *Find the product of* $1{\cdot}5^2 \times 0{\cdot}7854 \times 3$.

Opposite 1·5 on scale D is found on scale A the square of 1·5 or 2·25. It is not necessary to read off the value; arrange the line on the cursor coincident with 1·5, and the 1 on the slider also coincident with it; then opposite 0·7854 on the scale will be found the product of 0·7854 and 2·25. Finally, make the 1 on the slider coincident with the product referred to; then opposite 3 on the slider is the answer 5·3 on the scale.

Hence, $1{\cdot}5^2 \times 0{\cdot}7854 \times 3 = 5{\cdot}3$.

The converse operation is easily carried out.

EXAMPLE 5. *Given that the area of a circle is* 38·48 *sq. in., find its diameter.*

The diameter is obtained from $\sqrt{\frac{38{\cdot}48}{0{\cdot}7854}}$.

Set 0·7854 on the slider opposite to 38·48 on the scale A. Then opposite 1 on the slider is found, on the lower scale D, the diameter, 7 inches.

It should be noticed that the D scale may be used instead of the A scale and the result obtained with more accuracy.

Other exercises similar to those in the preceding examples may be obtained from any table of areas.

EXERCISES XXXVIII.

Use the slide rule to calculate the following:

1. Find the circumferences of circles whose diameters are (i) $\frac{1}{2}''$, (ii) $\frac{5}{8}''$, (iii) $2''$, (iv) $10''$, (v) $12{\cdot}75''$.

2. Find the areas of circles whose diameters are (i) $2''$, (ii) $2{\cdot}5''$, (iii) $3{\cdot}75''$, (iv) $4{\cdot}5''$, (v) $8\frac{3}{4}''$.

3. Given that the area of a circle in square inches is (i) 0·31, (ii) 2·4, (iii) 16·8, (iv) 70·88, find the diameter.

Use the slide rule to calculate the following :

4. Given that the circumference of a circle in inches is (i) 4·7, (ii) 7·1, (iii) 18·1, (iv) 20·4, find the diameter.

5. Find and tabulate the square roots of all the numbers 1, 2, 3, .. . 10.

6. Find and tabulate the reciprocals of all the numbers 1, 2, ..., 20.

SURFACE OF SIMPLE SOLIDS.

Prisms and Pyramids.—A **prism** is called a *right* prism when the edges of the solid are at right angles to the plane of the base. In a right prism the sides are rectangles. When the edges are not at right angles to the base the prism is called *oblique*.

A **pyramid** is a solid having a polygon for its base and all its side faces terminating in a point called the *vertex*. The

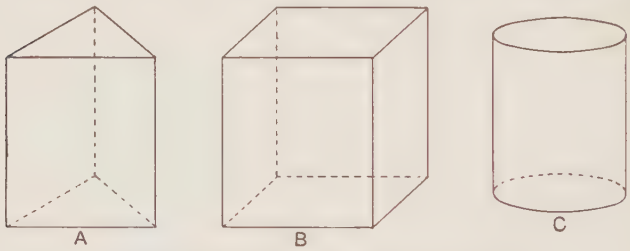


FIG. 85.—Triangular prism, square prism and cylinder.

base of a prism or pyramid may be any polygon, and the solid is called triangular, square, pentagonal, etc., according as the number of edges in the base is three, four, five, etc.

When the base of a pyramid is a circle the solid is called a **cone.**

A prism and a pyramid may be generated as follows :

If one extremity of a line be made to describe the sides of any polygon, then, if the line always remains vertical, the solid traced out will be a prism such as a **triangular prism** at *A*, Fig. 85, a **square prism** *B*, or a **cylinder** *C*.

If one extremity of a straight line always passes through a fixed point and the other describes the sides of any polygon, then a pyramid is formed such as a **triangular pyramid** at *D*, Fig. 86, a **square pyramid** at *E*, or a **cone** at *F*.

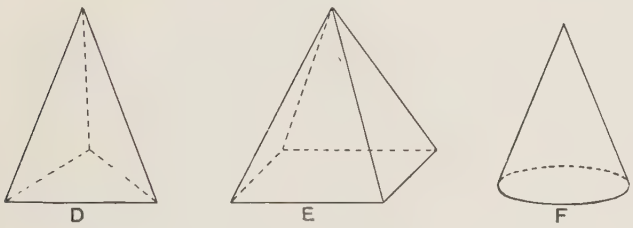


Fig 86.—A triangular pyramid, a square pyramid and a cone.

Surface of a Prism.—What is called a rectangular prism is the simplest and most familiar shape that a solid figure can take. Examples are furnished by a common brick, an ordinary box or cistern, and a four-sided room. The surface, which consists of six rectangles, is obtained in the same manner (p. 128) as the surface of a rectangular room which consists of the four walls, the floor and the ceiling.

When the length, breadth and thickness of a rectangular prism are equal the solid is called a **cube**, and the surface obviously consists of six equal squares.

Example 1. *The internal dimensions of a rectangular box without lid are: length 3 ft., width 2 ft., depth 1 ft. Find the interior surface and the cost of lining it with tin at* 1*s.* 6*d. per sq. ft.*

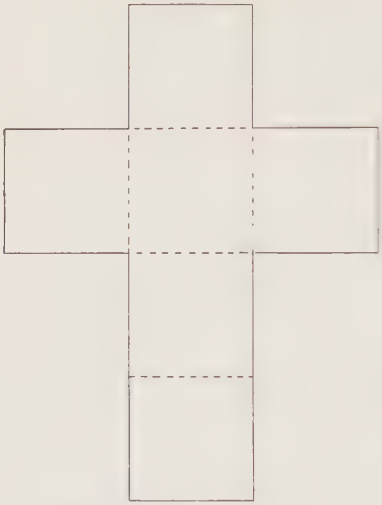
Surface of two sides $= 3 \times 1$ sq. ft.

Surface of two ends $= 2 \times 1$ sq. ft.

Surface of bottom $= 3 \times 2$ sq. ft.

$\therefore$ total surface $= 3 + 2 + 6 = 11$ sq. ft.

Cost $= 11 \times 1\frac{1}{2} = 16s.\ 6d.$



Example 2. *Find the surface of a cube, edge 2 ft.*

Area of each face

$=2\times2=4$ sq. ft.;

$\therefore$ surface of cube

$=4\times6=24$ sq. ft.

Draw on thick paper or thin cardboard six equal squares as in Fig. 87, make a cut halfway through the material along the dotted lines, and fold to make a cube.

Fig. 87.—Development of a cube.

Surface of a Cylinder.—The surface of a cylinder consists of two parts, the curved surface also called the **lateral** surface, and the areas of the two plane ends.

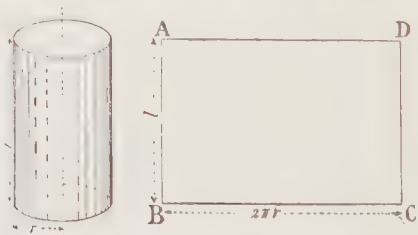


Fig. 88.—Lateral surface of a cylinder.

To obtain the area of the curved surface experimentally, cover a suitable wooden or metal cylinder with a piece of thin or squared paper, marking by two points the extremities of

the line where the edges come together. When the paper is unrolled and laid flat, a rectangle having one side equal to the circumference of the cylinder and the other equal to its height is obtained; and the number of units of area can be found either by calculation or by counting the enclosed squares.

EXAMPLE 1. *The diameter of a cylinder is* 4 *in., height* 20 *in.* (i) *What is its lateral surface,* (ii) *its total surface?*

(i) Lateral surface $= 3{\cdot}142 \times 4 \times 20 = 251{\cdot}4$ sq. in.

(ii) Area of two ends $= 2(0{\cdot}7854 \times 4^2) = 25{\cdot}13$ sq. in.;

$\therefore$ total surface $= 251{\cdot}4 + 25{\cdot}13 = 276{\cdot}53$ sq. in.

Surface of a Pyramid.—The surface of a pyramid consists of the area of the base and the sum of the areas of as many triangles as the figure has sides. Thus, the surface

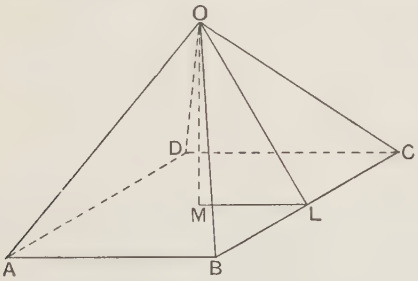


FIG. 89.—Surface of a pyramid.

of the square pyramid, Fig. 89, will consist of the area of the base $ABCD$ and the sum of the areas of the four equal triangles formed by the lines joining the points $ABCD$ to the vertex O.

The line LO joining the mid-point of the edge BC to the vertex O, and at right angles to BC, is called the **slant height**. The area of a face BCO is equal to half the product of a side of the base and the slant height, and the area of all the faces is equal to the product of half the sum of the sides

and the slant height. Hence the **lateral surface of a pyramid is the product of half the perimeter of the base and the slant height.** The **total surface** is the lateral surface together with the area of the base.

On a sheet of thick paper or thin cardboard draw a square *ABCD*, Fig. 90, and on each side draw equal triangles: make a half cut along the dotted lines and fold up into a model of a square pyramid.

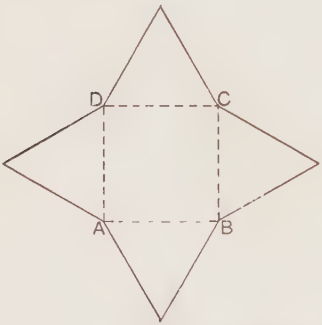


FIG. 90.—Development of a square pyramid.

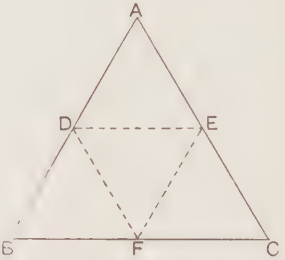


FIG. 91.—Development of a tetrahedron.

A pyramid on a triangular base may be obtained by drawing an equilateral triangle *ABC*, Fig. 91, and bisecting the sides at *DFE*; fold up along the dotted lines as before and a pyramid formed by four equal triangles is obtained called a **tetrahedron**.

EXAMPLE 1. *The base of a pyramid is a square of* 4 *ft. side, and the height of the pyramid is* 7 *ft. Find the total surface.*

In Fig. 89 $ML = 2$ ft., and $MO = 7$ ft.;

$$\therefore \text{ slant height} = \sqrt{2^2 + 7^2} = \sqrt{53}.$$

$$\text{Area of one face} = \tfrac{1}{2} \times 4\sqrt{53} = 2\sqrt{53};$$

$$\therefore \text{ area of four faces} = 8\sqrt{53}, \text{ area of base} = 16.$$

$$\text{Total surface} = 8\sqrt{53} + 16 = 74{\cdot}24 \text{ sq. ft.}$$

Surface of a Cone.—If the sloping surface of a cone

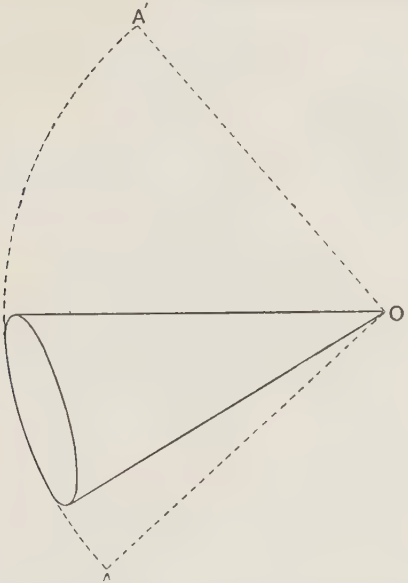


Fig. 92.—Development of a cone.

be covered by a piece of thin paper, then, when the paper is unrolled, a sector of a circle as OAA' Fig. 92 is obtained. Or, draw a straight line in ink from the vertex to the base of a cone, and before the ink is dry, roll the cone on a flat piece of white paper, the area of the sector, from the position in which it touches the paper to that in which it touches it a second time, will be the area or lateral surface of the cone.

If the radius of the base be R and height h (Fig. 93), half the perimeter of the base is πR and slant height is $\sqrt{R^2 + h^2}$.

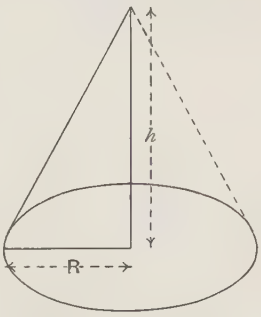


Fig. 93.—Surface of a cone.

EXAMPLE 2. *Find the total surface of a cone, base 6 ft. diameter, height 4 ft.*

As $r=3$ ft. and $h=4$; $\therefore$ slant height $=\sqrt{3^2+4^2}=5$.

Lateral surface $=\pi\times3\times5=47{\cdot}13$ sq. in.

Area of base $=0{\cdot}7854\times6^2=28{\cdot}28$ sq. in.

Total surface $=47{\cdot}13+28{\cdot}28=75{\cdot}41$ sq. in.

Sphere.—If a semicircle, *CDA*, Fig. 94, be assumed to revolve about a diameter such as *CD*, it will generate a solid

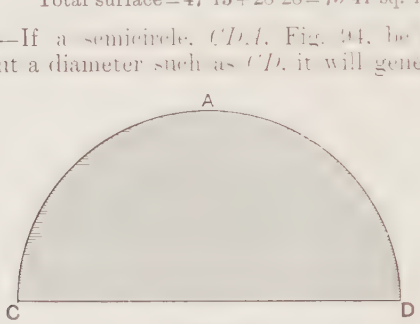


FIG. 94.

called a **sphere**. Similarly, a piece of very fine wire bent in the form of a semicircle *DAC* and made to revolve about a diameter *CD* would generate the surface of a sphere.

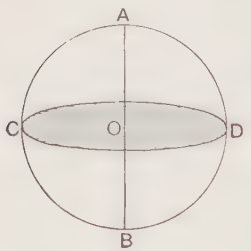


FIG. 95. Sphere.

Any line such as *CD* passing through the centre and terminated both ways by the surface of the sphere is a **diameter**.

Any plane such as *DOC* passing through the centre is a **great circle**. The radius *OC* or *OD* is called the **radius** of the sphere.

Surface of a Sphere.—The surface may be obtained from πd^2 or $4\pi r^2$, where d denotes the diameter, r the radius of the sphere and $\pi=3{\cdot}1416$ or $3{\cdot}142$.

Although the proof of this formula cannot be introduced here, there are many experimental methods by which the results may be verified.

One such method is to strip the cover from a tennis ball, and then to place the pieces on squared paper and trace the outline, the area may be found either by counting the enclosed squares, or by using Simpson's rule, p. 153.

If a balance is available the outline of the cover may be traced on cardboard and afterwards cut out and weighed. Using the same sheet of cardboard, the weight of a square inch, or square centimetre, can be obtained. Hence the area can be found and compared with that obtained by calculation.

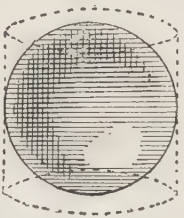


Fig. 96—Sphere and its circumscribing cylinder.

The surface of a sphere is equal to the *curved* surface of a cylinder, the diameter of the base of the cylinder being equal to the diameter of the sphere, and height equal to the diameter of the sphere. This cylinder (Fig. 96) is called the **circumscribing cylinder.**

Example 1. *The diameter of a sphere is* 2·53 *inches. Find its surface.*

$$\text{Surface} = 3{\cdot}142 \times 2{\cdot}53^2 = 20{\cdot}1 \text{ sq. in.}$$

$2 \log 2{\cdot}53 = 0{\cdot}8062$
$\log 3{\cdot}142 = 0{\cdot}4972$
$\overline{1{\cdot}3034}$ antilog $1{\cdot}3034 = 20{\cdot}1$.

Example 2. *The cost of gilding a sphere at* 6*d. per sq. in. is* £14. 8*s. Find the diameter of the sphere.*

Let d denote the diameter in inches. Then, as £14. 8*s.* = £14·4 we obtain

$$\frac{3{\cdot}142 \times d^2 \times 6}{240} = 14{\cdot}4\,;$$

$$\therefore\ d^2 = \frac{14{\cdot}4 \times 240}{3{\cdot}142 \times 6} = \frac{14{\cdot}4 \times 40}{3{\cdot}142}\,;$$

$$\therefore\ d = 13{\cdot}54 \text{ in.}$$

$\log 14{\cdot}4 = 1{\cdot}1584$
$\log 40 = 1{\cdot}6021$
$2{\cdot}7605$
$\log 3{\cdot}142 = 0{\cdot}4972$ antilog $1{\cdot}1316 = 13{\cdot}54$.
$2 \mid \overline{2{\cdot}2633}$
$1{\cdot}1316$

EXERCISES XXXIX.

1. Find the lateral and the total surface of a cylinder 10·75 ft. diameter, height 10 ft.

2. Find in square feet the lateral and total surface of a cylinder, radius of base 1 ft. 9 in., height 2 ft. 3 in.

3. The lateral surface of a cylinder is 400 sq. ft. If the height is 12 ft., find the diameter.

4. Find the lateral and total surface of a square pyramid, side of base 4 ft., height 7 ft.

5. The diameter of the base of a cone is 4 ft., height 7 ft. Find the total surface.

6. The lateral surface of a square pyramid is 97·3 sq. ft. If the side of the base is 4 ft., what is the slant height?

7. The slant side of a cone is 145 cm., the base is 174 cm. diameter. Find the total surface.

8. The lateral surface of a cone is 252·2 sq. ft. If the diameter of the base is 174 in., what is the length of the slant side of the cone?

9. The radius of the base of a cone is 3 m., height 60 dcm. Find its lateral surface.

10. The diameter of a sphere is 5·1 cm. Find its surface.

11. If the surface of a sphere is 300 sq. cm., what is its diameter?

12. The diameter of a sphere is 7·85 in. Find the surface in square feet.

13. The surface of a sphere is 154 sq. in. What is its diameter?

14. The radius of a sphere is 25½ ft. Find its surface. Also find the radius of a sphere whose surface is one-quarter the former.

15. Find the diameter of a circle such that its area will be equal to the surface of a sphere 2 in. diameter.

16. The lateral surface of a cone is 200 square feet. If the radius of the base is 5 ft., find (i) the slant height, (ii) the vertical height of the cone.

CHAPTER VIII.

Measurement of Volume ; Prism, Cube, Cylinder, Hollow Cylinder.

Measurement of Volume.—It has already been found, p. 62, that the volume of a solid may be found by multiplying together the three linear dimensions of length, breadth and thickness, all the dimensions being expressed in the same units.

Two of the dimensions are equal when the solid has a square or circular base.

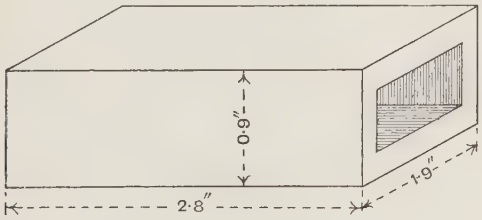


Fig. 97.—Volume of a hollow prism.

To obtain the volume of a hollow prism (Fig. 97), it is necessary to subtract the inner volume from that of the outer.

The weight of a solid may be found by multiplying the volume by the weight of unit volume of the material, or by multiplying the volume by the specific gravity of the material and by the weight of unit volume of water (p. 82).

EXAMPLE 1. *A hollow rectangular prism (similar in shape to a common match box) has its external dimensions 2·8, 1·9 and 0·9 in. respectively. If the thickness of the material is 0·2 in., find the volume of the solid. Also find its weight if the specific gravity of the material is* 0·56.

The interior dimensions will be 2·8, 1·5 and 0·5.

$$\text{External volume} = 2{\cdot}8 \times 1{\cdot}9 \times 0{\cdot}9 = 4{\cdot}788 \text{ cub. in.}$$

$$\text{Internal volume} = 2{\cdot}8 \times 1{\cdot}5 \times 0{\cdot}5 = 2{\cdot}1 \text{ cub. in.}$$

The arithmetical work is more easily carried out by writing the above in the form

$$\text{Volume} = 2{\cdot}8(1{\cdot}9 \times 0{\cdot}9 - 1{\cdot}5 \times 0{\cdot}5)$$
$$= 2{\cdot}8 \times 0{\cdot}96 = 2{\cdot}688 \text{ cub. in.}$$
$$\text{Weight} = 2{\cdot}688 \times 0{\cdot}56 \times 0{\cdot}036 = 0{\cdot}054 \text{ lb.}$$

Cube.—When the length, breadth and height are all equal the solid is called a **cube** and its volume is the cube of the number which denotes the length of one edge.

Conversely, the cube root of the number denoting the volume of a cube gives the length of its edge.

EXAMPLE 2. *The length of the edge of a cube is* 13·25 *in. What is the volume?*

$$\text{Volume} = (13{\cdot}25)^3 = 2326 \text{ cub. in.}$$
$$3 \log 13{\cdot}25 = 3 \times 1{\cdot}1222 = 3{\cdot}3666$$
$$\text{antilog } 3{\cdot}3666 = 2326.$$

EXAMPLE 3. *The volume of a cubical block is* 5832 *cub. ft. What is the length of an edge?*

$$\text{Length of edge} = \sqrt[3]{5832} = 18 \text{ ft.}$$
$$\log 5832 \div 3 = 3{\cdot}7658 \div 3 = 1{\cdot}2552$$
$$\text{antilog } 1{\cdot}2552 = 18.$$

EXERCISES XL.

[*A table of specific gravity and the weight of a cubic inch of various materials is given on* p. 83.]

1. Find the weight of the deck of a ship made of pine $1\frac{3}{4}$ in. thick that covers an area of 360 square feet. [Specific gravity 0·54.]

2. Three uniform sheets of lead each 1 sq. ft. in area are found to weigh 7·4 lb., 6 lb. and 5 lb. respectively. Find in each case the thickness of the lead sheets.

3. Find the thickness of steel plate when the weight per square foot is (i) 46 lb., (ii) 22 lb., (iii) 5 lb.

4. What is the weight in kilograms of a cubical block of metal, length of edge 1·02 metres, if 1 c.c. weighs 8·95 grams? Find the length of the diagonal of a face of the block.

5. A steel armour-plate is 15 ft. 3 in. long, 3 ft. 6 in. wide and $4\frac{1}{2}$ in. thick. Find its weight in tons.

6. A teak deck plank is 24 ft. long and $4\frac{1}{2}$ in. thick, 9 in. wide at one end and tapers gradually to 7 in. at the other end. Find its weight. [Specific gravity 0·98.]

7. The surface of the water in a reservoir 24 ft. 8 in. long, 12 ft. 9 in. wide and 9 ft. deep is raised 2·2 ft. when a cubical block of stone is placed in the reservoir. Find the length of edge of the stone.

8. What weight of 6 lb. lead will be required to cover the sides and bottom of a rectangular cistern 10 ft. long, $6\frac{1}{2}$ ft. wide and 7 ft. deep? What weight of water will the cistern hold?

9. What is the thickness of a rectangular piece of timber if the length is 10 ft. 6 in., the width 4 in. and the volume 1008 cub. in.?

10. A rectangular tank is 13 ft. 6 in. long and 9 ft. 9 in. wide. How much will the surface be lowered when 4·87 cub. yd. of water are drawn off?

11. A cistern 5 ft. long and 2 ft. wide is found to contain 4984 lb. of water. What is the height of the water?

12. The length of the edge of a cube of stone is 7·5 ft. Find its volume and weight. [Specific gravity 2·56.]

13. The external dimensions of a box without a lid made of wood one inch thick are: length 30 in., width 21 in. and depth 16 in. Find its internal volume.

14. The edge of a cube is 5 in. and its weight is $32\frac{1}{2}$ lb. What will be the weight of a cube of the same material the edge of which is 2 inches?

15. The base of a prism is a square of 3 ft. side, height 4 ft. The base of another prism is a square of 5 ft. side and its height is 9 ft. Compare the volumes and surfaces of the two prisms.
If the weight of the first is 1440 lb., what is the weight of the second, the material being the same in each case? What is the weight of a cubic foot of the material?

16. A schoolroom is 80 ft. long, 20 ft. wide and 10 ft. high. How many children will it hold if 80 cub. ft. are allowed for each child?

17. What is the weight of a rectangular log of wood 12 ft. 6 in. by 2 ft. 9 in. by 2 ft. 7 in., weighing 36 lb. per cub. ft.?

18. The external dimensions of a deal box are: length 4·5 ft., width 3 ft. and depth 2 ft. If the thickness is 2 in. and the specific gravity of the material 0·53, what is its weight?

19. The weight of a bar of iron, length 4 ft., sectional area 2·75 sq. in. is 37 lb. Find the weight of a bar 6·25 ft. long, sectional area 6·5 sq. in.

20. A rectangular vessel measures 4·25 metres by 35 decimetres by 48 centimetres. How many litres of water will it contain? If filled with a liquid, specific gravity 1·85, what weight of liquid will it hold?

21. From a reservoir of water 33·47 tons of water are drawn off. If the length of the reservoir is 25 ft. 4 in. and the width 6 ft. 4 in., how much will the surface be lowered?

22. The lead for a roof 48 ft. long and 32 ft. wide weighs 7593 lb. What is the thickness of the lead?

23. The internal dimensions of a tank are: length 1·05 m. width 2·25 m., depth 2·75 m. Find the number of litres and weight of water when the tank is full.

24. A section of a stream is 10 ft. wide and 10 in. deep. The mean flow of the water through the section is 3 miles per hour. Find how many gallons of water flow through the section in one hour.

25. A wrought-iron plate is 12 ft. 6 in. long and 5 ft. 3 in. broad. Its thickness tapers from 9 in. at its upper edge to 5 in. at its lower edge. Find its volume in cubic feet and its weight in tons.

26. The external dimensions of a tank are 6 ft. × 4 ft. × 4 ft. The mean thickness of the metal is $\frac{1}{4}$ in. What weight of water will it hold?

27. A weight of 85 tons placed on a certain vessel is observed to sink her $3\frac{1}{2}$ inches. Find the area of the section of the vessel made by the plane of the water.

28. The length of a rectangular reservoir is 12 ft. 3 in. and the width 8 ft. 6 in. If the volume is 324·6 cub. ft., find the depth. What is the depth of a cubical reservoir of the same volume?

29. The half-ordinates of a steel deck are 18 feet apart and their lengths are 0·4, 4·4, 10·7, 15·1, 16·8, 16·1, 12·8, 6·3 and 0·5 feet, respectively. Supposing the deck plating to be $\frac{5}{16}$ in. thick, find the area and calculate its weight, after allowing for two hatchways 18 ft. by 10 ft., and four openings $3\frac{1}{2}$ ft. by 3 ft.

30. A rectangular tank is 4 ft. long and 4 ft. wide. What number of gallons of water per foot of depth will it contain, if a cubic foot of water weighs 62·5 lb.?

31. A coal bunker is 20 ft. long, 12 ft. wide, and 7 ft. deep. How many tons of coal will it hold? [Allow 45 cubic feet to 1 ton.]

32. The dimensions of the different scantlings in a king-post roof truss (Fig. 98) are as follows: one tie beam, AB, 22 ft. $\times 9\frac{1}{2}$ in. $\times 5$ in., one king-post, DF, 7 ft. $\times 5$ in. $\times 5$ in., two principal rafters, AF and BF, each 13 ft. $\times 5$ in. $\times 5$ in., two struts, DB and DH, each 6 ft. $\times 2\frac{1}{2}$ in. $\times 5$ in. How many cubic feet of timber will the truss contain, and what will be the cost at 2*s.* per cub. ft.?

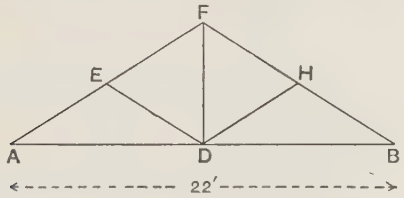


FIG. 98.—King-post roof truss.

33. A railway cutting is 14 ft. wide at the bottom, 26 ft. wide at the top, and the depth is 20 ft. Find the volume of earth in excavating a length of 50 yards.

34. A box open at the top has externally the form of a cube, each edge of which is 20 in. long. It is made up of five plates, each an inch thick. Find the volume of the material in cubic inches, and show that it is nearly three-tenths of the internal volume of the box.

Cylinder.—If r denotes the radius and d the diameter of a cylinder, then the area of the base is πr^2 or $0{\cdot}7854d^2$ and the volume is $\pi r^2 h$ or $0{\cdot}7854d^2 \times h$. Where h is the altitude or height of the cylinder.

EXAMPLE 1. *The diameter of a cast-iron cylinder is* 4 *in.; the height* 10 *in. Find the volume.*

$$\text{Vol.} = 0{\cdot}7854 \times 4^2 \times 10 = 125{\cdot}7 \text{ cub. in.}$$

$$\log 0{\cdot}7854 = \bar{1}{\cdot}8951$$
$$\log 160 = 2{\cdot}2041$$
$$2{\cdot}0992 \qquad \text{antilog } 2{\cdot}0992 = 125{\cdot}7.$$

$$\text{Weight} = 125{\cdot}7 \times 0{\cdot}26 = 32{\cdot}68 \text{ lb.}$$

EXAMPLE 2. *The volume of a cylinder is 907·6 cub. ft. If the height of the cylinder is 10 ft., find the diameter of the base.*

Let d denote the diameter.

$$\text{Vol.} = 0{\cdot}7854 \times d^2 \times 10 = 907{\cdot}6\ ;$$

$$\therefore\ d = \sqrt{\frac{907{\cdot}6}{7{\cdot}854}} = 10{\cdot}75 \text{ ft.}$$

$\log 907{\cdot}6 = 2{\cdot}9579$
$\log 7{\cdot}854 = 0{\cdot}8951$ antilog $1{\cdot}0314 = 10{\cdot}75$.
$2 \mid \overline{2{\cdot}0628}$
$\overline{1{\cdot}0314}$

EXAMPLE 3. *A coil of steel wire 600 yards long weighs 85 lb. Find the diameter of the wire.* [1 cub. in. of steel weighs 0·29 lb.]

Length of wire in inches $= 600 \times 3 \times 12$.

If d denotes the diameter in inches ;

$$\text{volume} = 0{\cdot}7854 \times d^2 \times 600 \times 36.$$

and $\text{weight} = 0{\cdot}7854 \times d^2 \times 600 \times 36 \times 0{\cdot}29 = 85$ lb. ;

$$\therefore\ d = \sqrt{\frac{85}{0{\cdot}7854 \times 600 \times 36 \times 0{\cdot}29}} = 0{\cdot}1314 \text{ in.}$$

$\log 85 = 1{\cdot}9294$
$\underline{3{\cdot}6920}$
$\overline{2{\cdot}2374}$

and $\bar{2}{\cdot}2374 \div 2 = \bar{1}{\cdot}1187$.

antilog $\bar{1}{\cdot}1187 = 0{\cdot}1314$.

$\log 0{\cdot}7854 = 1{\cdot}8951$
$\log 600 = 2{\cdot}7782$
$\log 36 = 1{\cdot}5563$
$\log 0{\cdot}29 = \bar{1}{\cdot}4624$
$\underline{3{\cdot}6920}$

FIG. 99.—Hollow cylinder.

Cross-section.—Any section by a plane at right angles to the axis of a cylinder is a circle ; an oblique section determines an ellipse. Hence, in referring to a cylinder, or other prism, it is usual to give the diameter, or area of cross-section, at right angles to the axis.

Hollow Cylinder.—If D is the external, and d the internal diameter, of a hollow cylinder of height h,

$$\text{area of cross section} = 0{\cdot}7854D^2 - 0{\cdot}7854d^2,$$

$$\text{or}\quad 0{\cdot}7854(D^2 - d^2),$$

$$\text{and}\quad \text{volume} = 0{\cdot}7854(D^2 - d^2)h.$$

EXAMPLE 4. *The external diameter of a hollow cast-iron cylinder is* 12 *in. The thickness of the cylinder is* 1 *in. and height* 10 *in. Find* (i) *area of cross-section,* (ii) *volume,* (iii) *its weight.* [1 cub. in. of cast iron weighs 0·26 lb.]

Inner diameter $= 12 - 2 = 10$ in.

(i) Area $= 0{\cdot}7854(12^2 - 10^2) = 0{\cdot}7854(44) = 34{\cdot}56$ sq. in.

(ii) Volume $= 34{\cdot}56 \times 10 = 345{\cdot}6$ cub. in.

(iii) Weight $= 345{\cdot}6 \times 0{\cdot}26 = 89{\cdot}86$ lb.

EXERCISES XLI.

1. The external diameter of a cast-iron pipe is 12 in. and the internal diameter 11 in. Find the weight of a length of 8 ft. of this piping.

2. A cast-iron pipe is 10 in. external diameter, thickness of metal $\frac{3}{4}$ in. and length 7 ft. Find its weight.

3. A hollow steel shaft has its external and internal diameters 14 in. and 10 in. respectively. Find the weight of 20 ft. length of shaft.

4. The external diameter of a cast-iron muff coupling is 8 in. A hole is bored through it of 4 in. diameter. Find the weight of the coupling if its length is 10 in.

5. What is the weight of a hollow steel pillar 10 ft. long, whose external diameter is 5 in. and internal diameter 4 in.? What is the diameter of a solid pillar of the same length and weight?

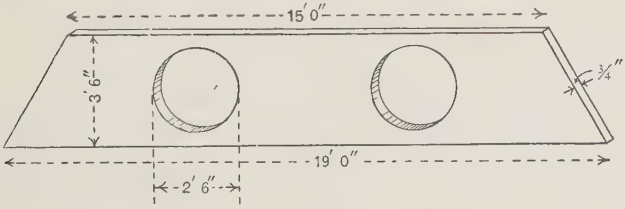


FIG. 100.

6. A steel plate is $\frac{3}{4}$ in. thick with two circular pieces 2 ft. 6 in. diameter cut from it and is of the form and dimensions Fig. 100. Find its weight.

7. A wrought-iron plate is 5 ft. 6 in. long, $\frac{3}{4}$ in. thick, $6\frac{1}{2}$ ft. wide at one end and 3 ft. at the other, and there is a hole 11 in. diameter cut through it as in Fig. 101. Find the volume and the weight of the plate.

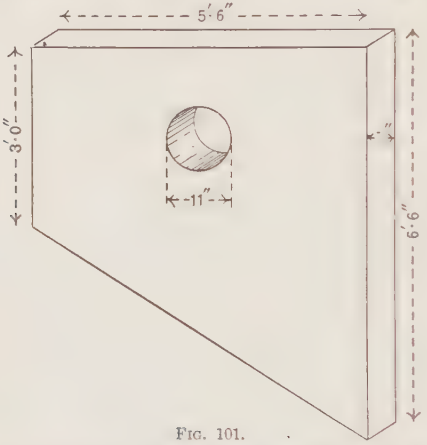


Fig. 101.

8. Two circular holes each $8\frac{1}{2}$ inches diameter are made in a steel plate of the form and dimensions shown in Fig. 102. Find the volume and the weight of the plate.

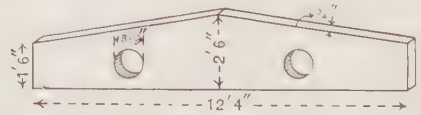


Fig. 102.

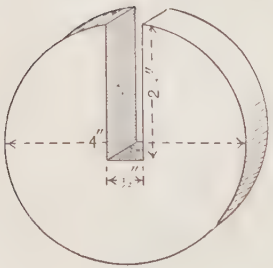


Fig. 103.

9. A set of cast-iron weights are required to be 4 in. diameter and to contain a slot $\frac{1}{2}$ in. wide and $2\frac{1}{4}$ in. long as in Fig. 103. Find in each case the thickness in inches when the weights are 1, 2, 3, 5 and 10 lb. respectively.

10. 1 c.c. of copper weighs 8·85 grams, 1 c.c. tin 7·29 grams, and 1 c.c. zinc 6·86 grams. Find the weight of a penny in grams, having given that the alloy consists of 95 % copper, 4 % tin and 1 % zinc, and the volume of a penny is 1·06 c.c.

11. A coil of brass wire of uniform circular cross-section 1 metre long weighs 19·6 grams in air and 17·2 grams in water. Find the specific gravity, and the diameter of the wire.

12. Find the weight of mercury required to fill a barometer tube, internal diameter 2 cm., length 120 cm., specific gravity of mercury 13·6.

13. A coil of wire 2 metres long of uniform circular cross-section, specific gravity 8·8, weighs 20 grams. Find its diameter.

14. A boiler contains 500 smoke tubes, each $2\frac{1}{4}$ in. internal diameter. Find the total area for draught through them.

What is the weight of one of the tubes if the thickness of metal is $\frac{1}{8}$ in., length 7 ft. and specific gravity of the material 7·8?

15. The dimensions of a teak rail, Fig. 104, are 8 in. by 3 in. It is rounded off in the form of a semicircle at each side. Find the weight of 20 ft. length. [Specific gravity of teak 0·98.]

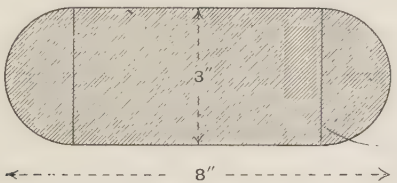


Fig. 104.

16. The external diameter of a pipe is 17 cm., its internal diameter is 11 cm. and length 5 metres. Find its weight in kilograms and in pounds, if the specific gravity of the material is 7·2.

17. Find the curved surface, the total surface, the volume and the weight of a brass cylinder, diameter of base 12 inches, height 10 inches. [1 cub. in. weighs 0·29 lb.]

18. The diameter of a pump bucket is 5 inches. If the column of water above the bucket is 28 feet, what is the weight of water on the bucket?

19. The diameter of the piston in a cylinder is 12 inches, and it moves through a certain distance each stroke. If the volume described is 1·57 cubic feet, find the distance.

20. The diameter of a cylindrical reservoir is 20 feet. Find the number of gallons poured in per minute, if the water in the cylinder rises at the rate of 2·45 ft. per minute.

21. The estimated cost of sinking a well 80 ft. deep and 4 ft. 6 in. diameter is £32. 19*s*. 6*d*. What is the cost per cubic yard?

22. Find the cost of 2 miles of cast-iron piping, 35 in. internal diameter, 1 in. thick. The price of iron is £9 per ton. Allow 5 per cent. extra for flanges.

23. The internal diameter of a hollow cast-iron roller is 20 in., its thickness $\frac{1}{2}$ in. and its length 3 ft. 6 in. Find the volume and weight.

24. If 3058 gallons of water are removed from a pond 25 ft. diameter, how far is the surface lowered?

25. Equal volumes of water, mercury and oil are contained in a cylindrical vessel 2 in. diameter and 8 in. in height. If the specific gravity of mercury be 13·59, that of oil 0·915, find the total weight of the water, mercury and oil.

26. A boiler contains 100 copper tubes each 2 in. outside diameter, 0·05 in. in thickness and 5 ft. 10 in. long. Find the total weight of the tubes.

27. Find the volume and weight of a cast-iron cylinder, radius of base 1 ft. 9 in., height 2 ft. 3 in.

28. A cylinder 4 ft. 6 in. diameter and 8 ft. long contains 6896 lb. of petroleum. Find the specific gravity of petroleum.

29. What will be the weight of a coil of steel wire 9778 yd. long and 0·025 in. diameter?

30. A circular metal disc has a radius of 14 ft. If a concentric disc is cut out of weight equal to the ring remaining, what is the radius of the inner disc?

31. Find the weight of an open wrought-iron cylinder 6 ft. long, the inner and outer diameters of which are 33 and 37 in. respectively.

32. Find to the nearest penny the expense of turfing a circular piece of ground 300 ft. in diameter at $2\frac{1}{2}d$. per sq. yd. Find also the cost of enclosing it at 9*d*. per ft.

33. A coil of wire, diameter 0·014 cm., specific gravity 8·8, weighs 10 grams. Find its length.

34. A path 6 ft. wide is made on the outside of a circular lawn 30 ft. diameter. What will be the cost of covering the path with gravel to an average depth of 3 in., if gravel costs 6*s*. per cub. yd.?

35. A piece of copper (specific gravity 8·9) 1 ft. long, 4 in. wide and $\frac{1}{2}$ in. thick, is drawn out into wire of uniform diameter $\frac{1}{16}$ in. Find the length in yd. and the weight of the wire.

36. A cylindrical gasometer with a flat top is required to hold two million cubic feet of gas. If the diameter is 200 ft., find the height.

37. Determine the weight of the gasometer in the preceding question, assuming the average weight of the iron plate and framing to be $7\frac{1}{2}$ lb. per sq. ft. of surface.

38. 20 gm. of mercury are poured into a vertical cylindrical tube and the height of the column is 13·5 cm. Find the diameter of the tube.

CHAPTER IX.

Volume of Pyramid. Frusta of Pyramid and Cone. Volume of Sphere. Similar Solids.

Volume of a Pyramid.—The volume of a pyramid is one-third the volume of the prism on the same base and of the same height. The proof of the rule cannot be given here, but it may be obtained by assuming the expression for the volume of a cube.

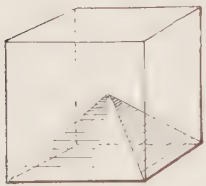


Fig. 105.

Thus, in Fig. 105 the base forms one of the six faces of a cube, and a pyramid is formed by joining each angular point to the centre of the cube. As each face of the cube may be used as base, the cube may be divided into six equal pyramids, the volume of each being one-sixth that of the cube.

Let the edge of the cube be 4 in.

$$\text{Volume of cube} = 4^3 = 64 \text{ cub. in.}$$

Volume of each pyramid $= \frac{1}{6}(16 \times 4) = \frac{1}{3}(16 \times 2)$.

Since the height of each pyramid is 2 ft., and area of base 16, we obtain the following rule:

Volume of pyramid $= \frac{1}{3}$ **(area of base)** $\times$ **(height).**

Several experimental methods of demonstration are available. Thus, a pyramid of metal may be placed in a graduated cylinder containing water, and the height before

and after immersion noted. The difference between the two readings is the volume required. Or the volume may be found by calculation and the weight from its known specific gravity; the result obtained can be verified by weighing the solid.

Using a small hollow prism, insert a pyramid having the same base and height (Fig. 106), fill up the prism with water, then, when the pyramid is removed the height of the water is found to be two-thirds that of the prism, or, the volume of the pyramid is one-third that of the prism.

EXAMPLE 1. *The base of a pyramid is a square of* 4 *ft. edge, and the height is* 7 *ft. Find its volume.*

$$\text{Area of base} = 4 \times 4 = 16 \text{ sq. ft.}$$

$$\text{Volume} = \tfrac{1}{3}(16 \times 7) = 37\tfrac{1}{3} \text{ cub. ft.}$$

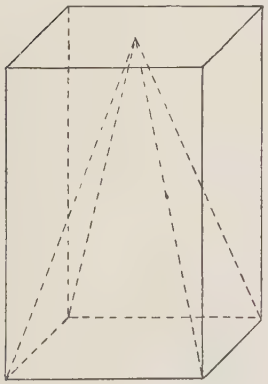


FIG. 106.—Prism and pyramid.

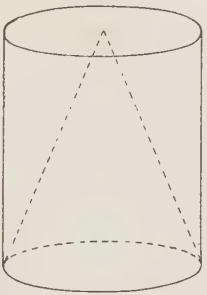


FIG. 107.—Cylinder and cone.

Cone.—If a cone be placed inside a hollow cylinder of same base and height (Fig. 107) and the cylinder be filled with water, then when the cone is removed the height of the water is two-thirds that of the cylinder; thus a cone has the same relationship to a cylinder that a pyramid has to the corresponding prism.

EXAMPLE 2. *The diameter of the base of a cone is 4 ft., height 7 ft. Find its volume.*

$$\text{Area of base} = 0{\cdot}7854 \times 4^2.$$

$$\text{Volume} = \tfrac{1}{3}(0{\cdot}7854 \times 4^2 \times 7) = 29{\cdot}32 \text{ cub. ft.}$$

$$\begin{aligned} \log 0{\cdot}7854 &= \bar{1}{\cdot}8951 \\ \log 16 &= 1{\cdot}2041 \\ \log 7 &= 0{\cdot}8451 \\ & \quad 1{\cdot}9443 \\ \log 3 &= 0{\cdot}4771 \\ & \quad 1{\cdot}4672 \end{aligned} \qquad \text{antilog } 1{\cdot}4672 = 29{\cdot}32.$$

Frustum of a Pyramid.—When a portion of a pyramid containing the vertex is removed the remaining portion is called a **frustum** of a pyramid.

In Fig. 108 a square pyramid is cut by a plane *EFGH* parallel to the base; the lower portion of the solid forms the

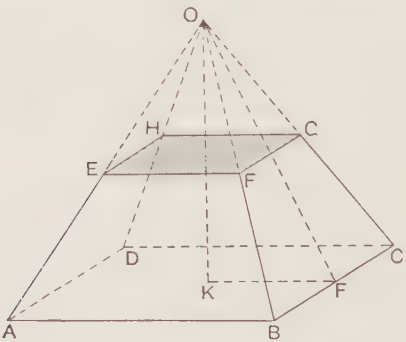


FIG. 108.—Frustum of pyramid.

frustum of a square pyramid. Its volume may be found by subtracting the volume of the upper, or smaller, pyramid of base *EFHG* and vertex *O*, from the whole pyramid of base *ABCD* and vertex *O*. Or, the volume may be found by the use of a formula.

Let A_1 denote the area of the base and A_2 that of the

face $EFGH$, h' the height of the frustum. Then the volume of the frustum is given by

$$V_1 = \frac{h'}{3}(A_1 + A_2 + \sqrt{A_1 A_2}).$$

EXAMPLE 3. *The edges of the parallel ends of the frustum of a square pyramid are* 40 *in. and* 20 *in. respectively; the height of the frustum is* 20 *in. Find its volume.*

As the edge AB is twice EF (Fig. 108) the height KO = twice the height of the frustum = 40 in. To obtain the volume of the frustum subtract the volume of the smaller pyramid from that of the larger.

$$\therefore \text{ volume} = \tfrac{40}{3} \times 40^2 - \tfrac{20}{3}(20)^2$$
$$= 21333{\cdot}3 - 2666{\cdot}6 = 18666{\cdot}7 \text{ cub. in.}$$

Using the formula for the volume of a frustum,

$$A_1 = 40, \quad A_2 = 20, \quad h' = 20;$$
$$\therefore \text{ volume} = \tfrac{20}{3}(40^2 + 20^2 + \sqrt{40^2 \times 20^2})$$
$$= \tfrac{20}{3}(2000 + 800) = 18666{\cdot}7 \text{ cub. in.}$$

Frustum of a Cone.—If a cone be cut by a plane EF parallel to the base AB the portion $ABFE$ is called a *frustum of a cone.*

As in the corresponding case of a pyramid, the volume of the frustum may be found by subtracting the volume of the upper, or smaller, cone of base EF and vertex O, from the whole cone of base AB and vertex O.

The volume may also be obtained by means of a formula as follows: Let R denote the radius of the base of the cone and r the radius of the smaller circle forming the upper face

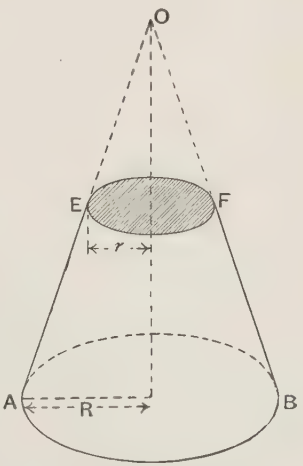


FIG. 109.—Frustum of a cone.

of the frustum, and at a distance d from the base: then, if V_1 denotes the volume of the frustum

$$V_1 = \frac{\pi d}{3}(R^2 + r^2 + Rr).$$

EXAMPLE 4. *The diameter of the base of a cone is 40 in.; the diameter of the upper circle is 20 in.; and the distance between them is 20 in. Find the volume of the frustum.*

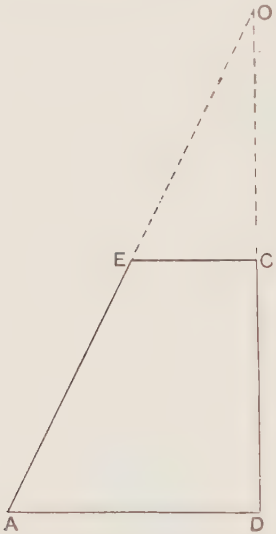


FIG. 110.

The radius of the base is 20 in. and the radius of the upper face 10 in.; hence, on squared paper draw two lines AD, DO at right angles to each other. Make DA (Fig. 110) equal to 20 in. and $DC = 20$ in. At C draw CE parallel to DA and equal to 10 in. Join A to E and produce the line AE to meet DC produced in O. Then a section of one-half the frustum and the cone of which it forms a part is obtained.

The volume of the frustum may be found by subtracting the volume of the smaller cone from that of the larger:

$\therefore$ volume of frustum

$$= \frac{\pi \times 20^2 \times 40}{3} - \frac{\pi \times 10^2 \times 20}{3}$$

$$= \frac{14000\pi}{3}.$$

Or, using the rule,

$$\text{volume} = \frac{\pi \times 20}{3}(20^2 + 10^2 + 2 \times 10)$$

$$= \frac{14000\pi}{3} = 14662{\cdot}7 \text{ cub. in.}$$

EXERCISES XLII.

[Specific gravity and weight of unit volume are given in Table I. p. 83.]

1. The base of a pyramid is a square of 6 in. side and the height is 8 in. Find its volume.

2. The base of a pyramid is a square of 4 ft. side and height 12 ft. Find the total area and volume of the pyramid.

3. The diameter of the base of a cast-iron cone is 7 in. and the height 10 in. What is the volume and weight of the cone? What would be the weight of a lead cone of the same dimensions?

4. The weight of a cast-iron cone is 6·5 lb. If the height of the cone is 6 in., find the diameter of its base.

5. A conical vessel, internal diameter 7 in., is required to hold 3·2 lb. of petroleum. What must be its height? [Specific gravity of petroleum = 0·87.]

6. The length of the slant side of a cone is 145 in. If the diameter of the base is 174 in., find the volume of the cone.

7. A conical glass, 2 in. diameter and $2\frac{1}{2}$ in. deep, is half full of water. Find the quantity of water it contains.

8. The radius of the base of a cone is 3 ft. and height 6 ft. Find its volume.

9. The base of a pyramid is a regular hexagon, edge of base 3 ft., height 6 ft. Find its volume.

10. The base of a pyramid is a hexagon of 2 ft. side. If the volume is 31·18 cub. ft., what is the height of the pyramid?

Volume of a Sphere.—The volume of a sphere varies as the cube of its diameter;

$$\therefore\ V \propto d^3 \quad \text{or} \quad V = kd^3, \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

where V denotes the volume, d the diameter of the sphere and k is a constant.

Several experimental proofs may be used to verify the rule. One such method is to measure, as accurately as possible, the diameter of a ball and to place it in a graduated cylinder in which there is sufficient water to ensure that the sphere can be completely immersed; the height of the surface of the water before and after immersion is noted and the difference is a volume equal to that of the sphere. In this manner the value of V is ascertained, and as d is known it is only necessary to substitute the values in Eq. (i) and the value of k can be obtained. Having obtained the numerical value of the constant k, then for any value of d the value of V can be found; and, conversely, the value of d when V is known.

The formula for the volume of a sphere may be obtained from the known value of the surface. Thus, if four points close together on the surface of a sphere be joined to the centre (Fig. 111), a small pyramid will be formed; the volume of such a pyramid will be $\frac{1}{3}$(area of base) × (height).

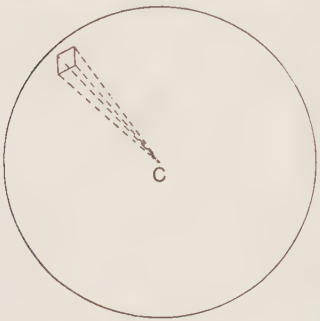


Fig. 111.—Volume of a sphere.

If a very large number of such pyramids be assumed, so that the bases occupy the whole surface of the sphere, then the sum of all the bases will be the surface of the sphere, and the volume of the sphere will be the sum of all the volumes of all the pyramids.

As the height of each pyramid is r, or $\frac{d}{2}$, where d is the diameter of sphere,

$$\text{volume of sphere} = \frac{1}{3}(\text{area of base}) \times \frac{d}{2}$$

$$= \frac{1}{3}(\text{surface area of sphere}) \times \frac{d}{2}$$

$$= \frac{1}{3}(\pi d^2) \times \frac{d}{2} = \frac{\pi}{6} d^3.$$

Example 1. *The diameter of a wooden sphere is 5·1 cm., and when immersed in a graduated cylinder it is found to displace 69 c.c. of water. Find the numerical value of the constant k in $V = kd^3$.*

Substituting the given values we obtain

$$69 = k \times (5{\cdot}1)^3, \text{ or } k = \frac{69}{(5{\cdot}1)^3} = 0{\cdot}52.$$

$$\log 69 = 1{\cdot}8388$$
$$3 \log 5{\cdot}1 = 2{\cdot}1228$$
$$\bar{1}{\cdot}7160 \qquad \text{antilog } \bar{1}{\cdot}7160 = 0{\cdot}52.$$

The more accurate value is 0·5236 or $\frac{\pi}{6}$.

Or, $$V = \frac{\pi}{6} d^3.$$

EXAMPLE 2. *A wooden ball is placed in a graduated cylinder, the volume of the water before immersion is* 100 *c.c. and the reading after immersion is* 230·8 *c.c. Find the diameter of the sphere.*

The volume of water displaced is $230{\cdot}8 - 100 = 130{\cdot}8$ c.c., and as this is the volume of the sphere, we obtain

$$130{\cdot}8 = 0{\cdot}5236d^3;$$

$$\therefore\ d = \sqrt[3]{\frac{130{\cdot}8}{0{\cdot}5236}} = 6{\cdot}296 \text{ cm.}$$

$\log 130{\cdot}8 = 2{\cdot}1165$
$\log 0{\cdot}5236 = \bar{1}{\cdot}7190$
$3 \mid 2{\cdot}3975$ antilog $0{\cdot}7991 = 6{\cdot}296.$
$0{\cdot}7991$

EXAMPLE 3. *Find the surface, volume and weight of a cast-iron sphere* 10·25 *in. diameter.*

$\text{Surface} = 3{\cdot}142 \times 10{\cdot}25^2 = 330{\cdot}1$ sq. in.
$\text{Volume} = 0{\cdot}5236 \times 10{\cdot}25^3 = 563{\cdot}7$ cub. in.
$\text{Weight} = 0{\cdot}5236 \times 10{\cdot}25^3 \times 0{\cdot}26 = 146{\cdot}6$ lb.

$2 \log 10{\cdot}25 = 2{\cdot}0204$
$\log 3{\cdot}142 = 0{\cdot}4972$ antilog $2{\cdot}5176 = 329{\cdot}4$
$2{\cdot}5176$

$3 \log 10{\cdot}25 = 3{\cdot}0321$
$\log 0{\cdot}5236 = \bar{1}{\cdot}7190$ antilog $2{\cdot}7511 = 567{\cdot}3$
$2{\cdot}7511$

$\text{Weight} = 563{\cdot}7 \times 0{\cdot}26 = 146{\cdot}6.$

EXAMPLE 4. *Find the volume and weight of a cast-iron shell* 10 *in. outside diameter and* 1 *in. thick.*

$\text{Internal diameter} = 10 - 2 = 8$ in.
$\text{Volume} = 0{\cdot}5236(10^3 - 8^3) = 0{\cdot}5236 \times 488$
$= 255{\cdot}5$ cub. in.

$\log 488 = 2{\cdot}6884$
$\log 0{\cdot}5236 = \bar{1}{\cdot}7190$ antilog $2{\cdot}4074 = 255{\cdot}5$
$2{\cdot}4074$

Similar Solids.—Two solids are said to be **similar** when they have the same shape or form, and the dimensions are either the same, or all the dimensions of one are greater or less than those of the other by equal amounts. Thus, all spheres and cubes are similar solids.

As a simple example, consider two prisms: in one the length, breadth and thickness are 4 in., 2 in. and 3 in. respectively, and in the other 12 in., 6 in. and 9 in.: or each edge, or linear dimension, of one is three times that of the other.

A drawing of the first made to any scale may be used for the second by using a scale three times the former.

It is easy to verify that as each linear dimension in the second is three times that of the first, the area of each face is 9 times the first, and the volume of the second solid is 27 times the first.

Thus, the area of a face of one is 4×2 in. $= 8$ sq. in.

And area of the corresponding face of the other is

$$12 \times 6 = 72 \text{ sq. in.};$$

$$\therefore \quad 72 \div 8 = 9.$$

$$\text{Volume of first solid} = 4 \times 2 \times 3 = 24 \text{ cub. in.}$$

$$\text{Volume of second solid} = 12 \times 6 \times 9 = 648 \text{ cub. in.}$$

$$\text{Also} \quad 648 \div 24 = 27.$$

EXAMPLE 5. *The lengths of the edges of two cubes are 2 and 4 in. respectively. Compare the surfaces and volumes of the two solids. If the weight of the first is 2·24 lb., what is the weight of the second?*

The linear dimensions or the ratio of the length of the edges are as 4 to 2, or 2 : 1.

The surface of the first will be $6 \times 2^2 = 16$ sq. in.

The surface of the second will be $6 \times 4^2 = 64$ sq. in.

Ratio of surfaces $= 64 : 16 = 4 : 1$ or $2^2 : 1$.

The volumes are as 4^3 to 2^3 or $8 : 1 = 2^3 : 1$.

As the weight of the first is 2·24 lb., the weight of the second is

$$8 \times 2{\cdot}24 = 17{\cdot}92 \text{ lb.}$$

The weight of the second could also be obtained in a more general manner as follows: Let w denote the weight of a cubic inch of the material.

Then, weight of first cube $2{\cdot}24 = 2^3 \times w$;

$$\therefore \quad w = \frac{2{\cdot}24}{8}$$

$$\text{Weight of second cube} = 4^3 \times \frac{2{\cdot}24}{8} = 17{\cdot}92 \text{ lb.}$$

EXERCISES XLIII.

1. Find the volume and weight of a cast-iron ball 7 in. diameter.

2. The diameter of a stone sphere is 12 in. Find its weight if its specific gravity is 2·74.

3. The external diameter of a hollow cast-iron spherical ball is 22 in., its internal diameter 18 in. Find its weight.

4. The external diameter of a spherical shell is 8 in. and the internal diameter 7·2 in. Find its weight if the specific gravity of the material is 7·2.

Find the diameter of a solid sphere of the same material and of the same weight.

5. Find the internal diameter of a hemispherical cup to hold half a pint of water.

6. The external diameter of a hollow copper sphere is 8 in. Find its weight if the thickness is (i) $\frac{1}{10}$ in., (ii) $\frac{1}{20}$ in., (iii) also find the weight in each case, using the approximate formula

$$\text{volume} = (\text{surface of sphere}) \times (\text{thickness}).$$

Find the percentage error. [1 cub. in. of copper weighs 0·319 lb.]

7. The internal diameter of a hollow hemispherical basin is 10·19 inches. Find the quantity of water it will hold when full.

8. If 30 cub. in. of powder weigh 1 lb., find the weight of powder required to fill a shell whose internal diameter is 8 in.

9. A spherical ball 15 in. diameter is melted down and cast into a conical mould. If the height of the cone is 16·88 in., what is the diameter of its base?

10. A spherical balloon of 10 metres diameter, made of thin material weighing 30 grams per square metre, is filled with coal gas, specific gravity compared with air 0·438. Calculate the weight that this balloon will lift, assuming 1 litre of air to weigh 1·28 grams.

11. Find the diameter of a sphere of lead weighing the same as a cast-iron sphere 13·56 in. diameter.

12. The outside diameter of a cast-iron spherical shell is 14 in. Find its internal diameter if its weight is 100 lb.

13. A bell in the form of a hemisphere is 2·6 in. internal diameter and $\frac{1}{8}$ in. thick. How much metal by volume is used in making it?

14. A cast-iron dumb-bell consists of two spheres each 5 in. diameter 20 in. apart (Fig. 112), connected by a cylindrical bar $\frac{3}{4}$ in. diameter. Find its volume and weight.

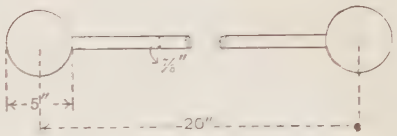


Fig. 112.

15. How many spherical lead shot $\frac{1}{4}$ in. diameter are there in a pound weight?

16. A wooden ball is 30 cm. diameter and weighs 7·6 kilograms. Find the weight of a cubic centimetre.

MISCELLANEOUS EXERCISES XLIV.

1. How much will a man have to pay for 10 cwt. of lead which he buys at 14 per cent. more than the wholesale price of 14s. 6d. per cwt.?

2. What is the area in square feet of a rectangle 55 cm. long and 35 cm. wide?

3. A circular lawn has an area of $\frac{1}{4}$ acre. What is its diameter? There is a path round the lawn 6 ft. wide. What is the area of the path?

4. The floor of a room 50 ft. 6 in. long is 25 ft. 4 in. wide at one end and 18 ft. 8 in. at the other. What is the area of the floor?

5. The sides of a rectangular plate measured to the nearest sixteenth of an inch are $13\frac{5}{16}$ and $17\frac{4}{16}$. Find the area of the plate.

6. The area of a rectangle is equal to that of a square, the length of whose side is 20·78 ft. If the length of the rectangle is 81 ft., what is its width?

7. The area of a circle is 42·23 square feet. Find its circumference.

8. The circumference (or girth) of a circle is 4 ft. Find its diameter.

9. The perimeters of a circle, a square and an equilateral triangle are each one foot. Find the area in each case.

10. Posts are set up round a circular track at a distance apart of 5 metres 75 centimetres. If there are 120 posts, what is the length of the track? How long will a person take to run round it at the rate of 1 kilometre in 3 minutes?

11. Two men measure a rectangular box; one finds its length, breadth and depth in inches to be 5·32, 4·15 and 3·29 in. respectively. The other finds them to be 5·35, 4·17 and 3·33 in. Calculate the volume in each case. What is the mean of the two? What is the percentage difference of each from the mean?

12. A kite consists of a semicircle and a triangle, Fig. 113. Find its surface.

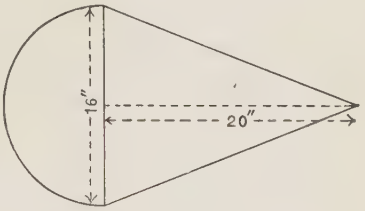


FIG. 113.

13. Find the area of

(i) A circle, radius 14 ft.

(ii) The curved surface of a right cone, height 10 ft., on a circular base 14 ft. radius.

(iii) A sphere, radius 14 ft.

14. Find the area of

(i) A triangle; base 15 ft., height 11 ft.

(ii) A triangle, with sides 39, 42 and 45 ft.

(iii) A circle, circumference 2 ft.

(iv) The curved surface of a cone, radius of base 3 ft., altitude 6 ft.

15. The length of an edge of a cubical block is 12·13 cm. Find the weight of the block in pounds if 1 c.c. of the material weighs 4·21 grams.

16. What is the weight in kilograms of a cube of metal, length of edge 2·26 metres. [Specific gravity = 7·2.]

17. If the specific gravity of teak is 0·98 and that of wrought iron 7·7, what thickness of iron will have the same weight per square foot as a plank of teak 4 inches thick?

18. The cross-section of a log of timber is 2 ft. $7\frac{1}{2}$ in. by 2 ft. 3 in. What length of this log will contain 315 cub. ft.?

19. A slab of copper 1 ft. long, 9 in. wide and $\frac{5}{8}$ in. thick is rolled into a sheet 6 ft. long and 4 ft. wide. Find the thickness of the sheet.

20. A square plate of iron, side one foot, thickness 1 in., weighs 37·4 lb. Find the weight of a circular disc of the same metal 2 ft. 6 in. diameter and $1\frac{1}{2}$ in. thick.

21. Find the surface of a ceiling 7 yd. 6 in. long, and 5 yd. 1 ft. wide. How many cubic feet of plaster will be required to plaster it an eighth of an inch thick?

22. A heavy sphere 25 cm. diameter is placed in a cylinder, height 3 dcm. and radius 1·5 dcm., and the cylinder is filled with water. What is the volume of water in the cylinder?

23. A cylinder, 20 feet long and 5 feet in diameter, is capped at one end by a hemisphere and at the other by a cone 6 feet in altitude. Find to the nearest penny the cost of painting the whole surface at 6*d.* per square yard.

24. A circular lake has an area of $\frac{1}{2}$ acre, and is covered with ice of an average thickness of 2 inches. Find the weight of the ice in tons, if a cubic foot of it weighs 56 lb. Find the cost of enclosing the lake by a fence costing 2*s.* 6*d.* per yard.

25. The external and internal diameters of a hollow steel cylindrical shaft are 16 and 8 inches respectively. Find the area of a cross-section.

If the specific gravity of the material is 7·8 and the length of the shaft 4 yards, find its weight.

26. The internal dimensions of a wrought-iron cylinder with hemispherical ends are: diameter 4 ft., total length 8 ft. How many cubic feet of water will it hold when full?

What is the weight of the metal forming the cylinder if the thickness is $\frac{1}{2}$ inch? [One cubic inch of wrought iron weighs 0·28 lb.]

27. The radius of the base of a cylinder is 8 in. and its height 4 ft. 2 in. Find its volume.

28. Find the total surface and volume of a cylinder, radius of base 1 ft. 8 in., height 2 ft. 4 in.

29. A hollow cylinder, 4 ft. internal diameter, contains water to a height of 1 ft. A heavy sphere, diameter 2 ft., is placed inside. What will now be the height of the water?

30. A cistern which contains 70 cub. ft. of water is emptied by a circular hole 1 in. diameter in 40 minutes. Find the mean speed of the water.

31. Find the area of one end of a house consisting of a rectangle $30'\ 9'' \times 20'\ 3''$ surmounted by a triangle base $20'\ 3''$, height 10 ft.

32. The external dimensions of a cistern are: length 6 ft. $4\frac{3}{4}$ in., width 4 ft. $3\frac{3}{4}$ in. and depth 5 ft. $6\frac{3}{4}$ in. The mean thickness of the material is $\frac{3}{4}$ in. What is the number of gallons of water which the cistern can hold?

33. Find the internal surface of the cistern in Question 32; also the cost of lining it with lead weighing 6 lb. per sq. ft. and costing 3*d.* per lb.

34. If a block of stone 14 ft. long, 4 ft. wide and 3 ft. thick weighs 20 tons 3 cwt., what will be the weight of a block 19 ft. long, 3 ft. wide, and 2 ft. thick?

35. Compare the quantities of air in two rooms, one 24 ft. 8 in. by 23 ft. 9 in. and 9 ft. high, the other 18 ft. 6 in. by 15 ft. 3 in. and 12 ft. 8 in. high.

36. A roof flat 48 ft. long and 32 ft. wide is to be covered with lead $1\frac{1}{2}$ inch thick. Allowing 5 per cent. for roll joints, find the weight of lead required. [1 cub. ft. of lead weighs 712 lb.]

37. A cube, length of edge 3·25 in., weighs 3·375 lb. Find the weight of a cube of the same material 4·875 in. edge.

38. Find the volumes of

(i) A sphere, radius 14 ft.

(ii) A cylinder 14 ft. long, radius of cross-section 8 ft.

39. An air current passes through an airway 10 ft. diameter at a velocity of 7 ft. per sec. What quantity of air passes per minute? If the diameter of the airway at a certain place is 12 ft., what will be the velocity of the air, the quantity remaining unchanged?

40. A cylindrical hole 3 in. diameter is bored through a cubical block of oak of 10 in. edge and at right angles to one of the faces of the cube. Find the weight of the block. [Specific gravity = 0·93.]

41. ABC is a triangle, C being a right angle. The side AB is 15·34 in.; the side BC is 10·15 in. What is the length of AC? What is the area of the triangle? If this is the shape of a piece of sheet brass 0·13 in. thick, and if brass weighs 0·3 lb. per cub. in., what is its weight?

42. A brass tube 8 ft. long has an outside diameter 3 in., inside diameter 2·8 in. What is the volume of the brass? If a cubic inch weighs 0·3 lb., what is the weight of the tube?

43. The height of a conical wine glass is equal to the diameter of the upper edge. If 500 drops of liquid are let fall into it, how high will the liquid stand in the glass, assuming each drop to be spherical and $\frac{1}{8}$ in. diameter?

44. A pipe has to discharge 300 gallons of water per minute. What should be its diameter if the velocity of the water is not to exceed 4 ft. per second?

45. Find the surface, volume and weight of a cast iron sphere 10·25 in. diameter.

46. The external diameter of a cast iron shell is 10 in. Find the thickness of the shell if its weight is 66·43 lb.

47. A 28 lb. weight is made in the form of the frustum of a square pyramid, edge of base 4·3 in., upper face 4 in. edge. Show that the height is 6·25 in.

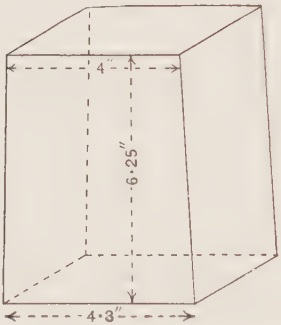


Fig. 114.—Frustum of a square pyramid

Fig. 115.—Frustum of a cone.

48. A 56 lb. weight is made in the form of the frustum of a square pyramid, edge of base 5·7 in., upper edge 5·2 in. Find its height.

49. A 56 lb. weight is made in the form of a frustum of a cone, diameter of base 5·7 in., upper face 5·2 in. Find the height (Fig. 115).

CHAPTER X.

Angular Measurement. Ratios of an Angle; Sine, Cosine, Tangent. Use of Tables.

Angular Measurement.—The angle between two intersecting lines may be estimated in degrees as on p. 30. Another common method called the **radian** measure of an angle depends upon the ratio formed by the radius of the circle and the arc subtended by the angle.

$$\text{Angle} = \frac{\text{length of arc}}{\text{length of radius}}.$$

An angle whose magnitude is unity is obtained when the length of the arc BC (Fig. 116) is equal to the radius AB. Such an angle is the **unit angle** and is called a **radian**.

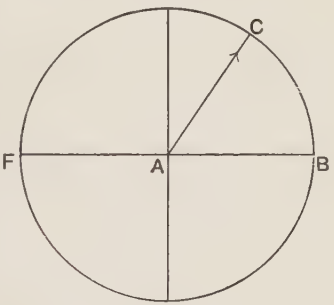


Fig. 116.

To set out an angle of one radian we may proceed as follows:

Describe a circle with any convenient length as radius and draw a horizontal diameter such as FAB (Fig. 117). Draw a straight line equal in length to the radius of the circle on a strip of tracing paper. Put one end of the line coincident with B, and a small portion of the line to coincide with the arc. Insert a needle point at that part of the line where it leaves the curve.

Rotate the tracing paper about this point as a centre until it coincides with another short length of the curve; in this

manner we eventually obtain a length of arc BC equal to the radius AB. Join the point C to the centre of the circle (Fig. 116). Measure the angle subtended by the arc; it will be found to be 57·3°.

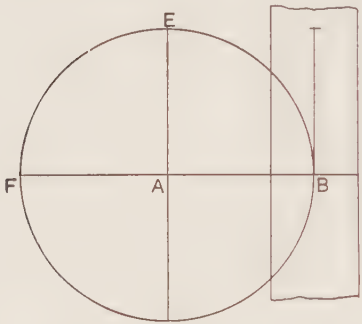


Fig. 117.

The result obtained may be verified by calculation. Thus, in moving from B to F, the point C (Fig. 116) describes an angle equal to 180°; if r is the radius of the circle, then the length of the semicircle is πr (p. 140);

$$\therefore \text{ angle} = \frac{\text{arc}}{\text{radius}} = \frac{\pi r}{r} = \pi.$$

Hence 180° and π radians both denote the same angle. Thus, a reading in degrees is converted into radian measure by multiplying the given number of degrees by π and dividing the product by 180.

Conversely, a given number of radians can be expressed in degrees by multiplying by 180 and dividing by π.

Example 1. *Find the number of radians in* 120°.

$$\text{Radians} = \frac{120\pi}{180} = \frac{2\pi}{3} = 2{\cdot}094.$$

Example 2. *Find the number of degrees in* 2·4 *radians.*

$$\text{Degrees} = \frac{2{\cdot}4 \times 180}{\pi} = 137{\cdot}5°.$$

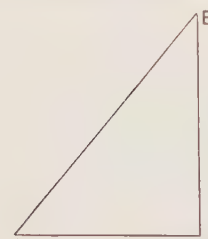


Fig. 118.

Ratios of an Angle.—Another method of estimating the magnitude of the angle between two lines which intersect at a point A (Fig. 118) is as follows:

From any point B in one line draw a line perpendicular to AC. This determines the three sides AC, CB and AB of a right-angled triangle.

The ratio $\frac{BC}{AB}$ or $\frac{\textit{opposite side}}{\textit{hypotenuse}}$ is the **sine** of A.

The ratio $\frac{AC}{AB}$ or $\frac{\textit{adjacent side}}{\textit{hypotenuse}}$ is the **cosine** of A.

The ratio $\frac{BC}{AC}$ or $\frac{\textit{adjacent side}}{\textit{opposite side}}$ is the **tangent** of A.

The preceding ratios are abbreviated into:

$$\sin A = \frac{BC}{AB}, \quad \cos A = \frac{AC}{AB}, \quad \tan A = \frac{BC}{AC}.$$

These three ratios together with three ratios formed by their reciprocals are called the **trigonometrical ratios.** The reciprocals are

$$\text{cosecant } A = \frac{AB}{BC}, \quad \text{secant } A = \frac{AB}{AC} \quad \text{and} \quad \text{cotangent } A = \frac{AC}{BC}.$$

These are abbreviated into

$$\operatorname{cosec} A = \frac{AB}{BC}, \quad \sec A = \frac{AB}{AC}, \quad \cot A = \frac{AC}{BC}.$$

As the hypotenuse is the greatest side in any right-angled triangle it follows that the sine and cosine of an angle which have the hypotenuse in the denominator cannot be greater than unity, the tangent may have any numerical value.

EXAMPLE 3. *Draw an angle of* 60° *and find by measurement and calculation its sine, cosine and tangent.*

Take any horizontal line as base and with a protractor make an angle $DAC = 60°$. Or by construction, with any point A as centre and radius 10 units describe an arc of a circle intersecting the line AD in D. With D as centre, and the same radius describe an arc intersecting the former in B. Join A to B. Then the angle BAC is an angle of 60°.

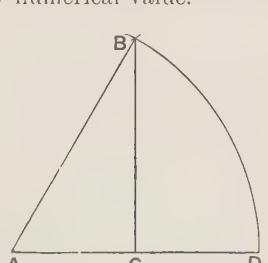


FIG. 119.—Construction of an angle of 60°.

Draw BC perpendicular to AD, and measure the length of $BC = 8{\cdot}6$ units, and $AC = 5$ units.

$$\therefore \sin 60^\circ = \frac{BC}{AB} = \frac{8{\cdot}6}{10} = 0{\cdot}86.$$

$$\cos 60^\circ = \frac{AC}{AB} = \frac{5}{10} = 0{\cdot}5.$$

$$\tan 60^\circ = \frac{BC}{AC} = \frac{8{\cdot}66}{5} = 1{\cdot}732.$$

Draw any line $B'C''$ parallel to BC as in Fig. 122, and read off the lengths of AB', $B'C''$ and $C''A$, it will be found that the numerical values of the ratios are the same as before. Thus, the magnitude of an angle is independent of the lengths of the lines forming its sides.

EXAMPLE 4. *Construct an angle whose sine is 0·6 and find the value of its cosine and tangent.*

Take any horizontal line AB on squared paper and at B draw BC at right angles to AB and equal to 6 units. With C as centre and radius 10 units describe an arc of a circle intersecting BA at A. Join C to A. Then BAC is the angle required; the length of AB by measurement is found to be 8 units.

$$\cos A = \tfrac{8}{10} = 0{\cdot}8, \quad \tan A = \tfrac{6}{8} = 0{\cdot}75.$$

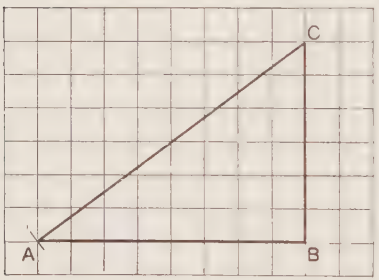


FIG. 120.—To construct an angle whose sine is 0·6.

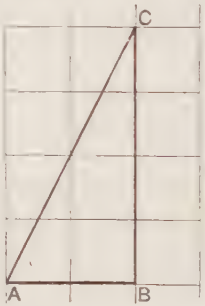


FIG. 121.—To construct an angle whose tangent is 2 units.

EXAMPLE 5. *Construct an angle whose tangent is 2 units, and find the sine and cosine of the angle.*

On squared paper take two lines BC vertical and AB horizontal, making $BC = 2$ units and $AB = 1$ unit. Join C to A. Then, by measurement, $AC = 2{\cdot}24$;

$$\therefore \sin A = \frac{2}{2{\cdot}24} = 0{\cdot}89, \quad \cos A = \frac{1}{2{\cdot}24} = 0{\cdot}45.$$

At the extreme left-hand side of Table IV. the angles 0° to 45° are found. This column and its continuation, 45° to 90°, at the extreme right of the page, together with the columns marked sine, tangent, cotangent and cosine, enable the numerical value of any of these ratios to be obtained for any angle from 0° to 90°.

Thus, for example, for an angle of 30° we find the following values:

Angle.	Radians.	Sine.	Tangent.	Cotangent.	Cosine.
30°	0·5236	0·5000	0·5774	1·7321	0·8660

The values of the ratios for angles greater than 45° are obtained by reading the angles in an upward direction, the names of the ratios, sine, tangent, etc. being marked along the bottom row. Thus, we find the following values for an angle of 60°.

0·5000	0·5774	1·7321	0·8660	1·0472	60°
Cosine.	Cotangent.	Tangent.	Sine.	Radians.	Angle.

EXERCISES XLV.

1. Draw an angle of 24° and find by measurement the values of its sine and cosine.

2. Draw an angle of 54° and find its sine and cosine.

3. The sine of an angle is 0·39. Find the magnitude of the angle and the cosine and tangent of the angle.

4. Construct an angle whose cosine is 0·81. Find the magnitude of the angle and the values of its sine and tangent.

5. Construct a right-angled triangle ABC, the hypotenuse $AB = 10$ cm., $\tan A = 0{\cdot}9$. Measure the angle A and find the sine and cosine of the angle.

6. Draw a triangle ABC having a right angle at C, hypotenuse $AB = 10$ units, and $\tan A = 0{\cdot}7$. Find the angle A and the values of $\sin A$ and $\cos A$.

Show that $\sin^2 A + \cos^2 A = 1$, and that $\tan A = \sin A \div \cos A$.

7. Construct a triangle ABC having given the side $BC = 4{\cdot}2''$, the angle $ABC = 71°$, the angle $BAC = 65{\cdot}6°$. Measure the angle ACB and the sides AB, AC.

8. Construct a triangle ABC to the following data :

$$AB = 3{\cdot}8'',\ BC = 3{\cdot}3'',\ \sin ABC = 0{\cdot}652.$$

Taking AB as the base, draw the perpendicular from the vertex C, and measure the altitude of the triangle. Verify the result by calculation, using the given sine.

What is the area of the triangle ?

What is the length of the base of a triangle of equal area, but having an altitude of $1{\cdot}6''$?

9. Draw a triangle ABC, base $BC = 10{\cdot}8$ cm., sides AB and $AC = 15{\cdot}7$ cm. and $8{\cdot}9$ cm. respectively. Draw a line parallel to the base bisecting the side AB in D, and intersecting the side AC at E. Find (in square inches) the area of the triangle ABC and the area of the trapezium $BCED$. What is the numerical value of the ratio of these areas ?

10. Draw a right-angled triangle ABC, base $BC = 5{\cdot}63$ in., hypotenuse 10 in. Measure the altitude CA and the base angle B : calculate the sine, cosine and tangent of B.

11. Measure the given angle BAC in degrees and decimals of a degree. Measure AC in inches and decimals of an inch. From C draw CB perpendicular to AC and measure CB and AB. Divide CB and AB by AC and give the quotient in each case. Draw $B'C'$ parallel to BC. Divide $C'B'$ and AB' by AC' and give the quotient in each case.

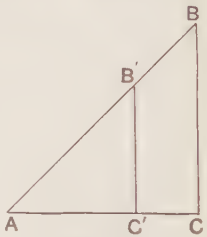


Fig. 122.

12. Measure in degrees the angle between the two given lines MM and LL. What is the magnitude of the angle in radians ?

Fig. 123.

13. The tangent of an angle is 0·633. Construct the angle. Determine the sine and cosine of the angle.

14. Measure carefully the given angle A in degrees and in radians, also find the sine and cosine of the angle (Fig. 118).

15. The cosine of an angle is 0·51. Construct the angle. Measure it in degrees and radians. What is the sine and tangent of the angle?

16. Draw two lines OA, OA each 10″ long, and containing an angle AOA of 88°. State the distance apart of A, A.

17. Draw two lines OX, OY at right angles. Mark a point P distant 5·25″ from OY and 3·81″ from OX. Join OP and measure the length of OP. Verify your answer by calculation.

Measure the angle XOP in degrees; verify this result by first calculating the tangent of the angle XOP and then referring to Table IV.

18. Draw a right-angled triangle ABC, base $BC = 7·66$″, hypotenuse $BA = 10$″. With centre B, radius BA, draw an arc to cut the base BC produced in D, and draw the chord AD.

Measure the base angle B, the altitude CA, the chord AD and the arc AD. Find the sine of the angle B and the radian measure of the angle. Verify by means of Table IV.

19. Construct a right-angled triangle ABC, base $BC = 10$ in., base angle 13°, C being a right angle. Measure the height CA. Verify by calculation, using the value of tan 13°, Table IV.

20. Construct the figure to the given dimensions. Measure in centimetres the length of OB (Fig. 124).

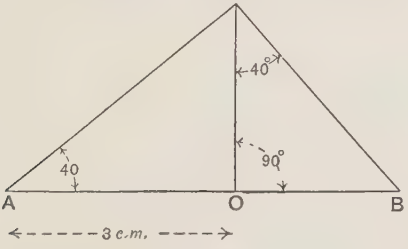


Fig. 124.

Obtain tan 40° from Table IV. Square and multiply the value by 3. Compare the result obtained by calculation with the previous measured value of OB.

21. Draw the triangle ABC on squared paper to the given dimensions. Draw a perpendicular from A on BC and find its length; hence, find the area of the triangle.

Measure the angle ABC and find the value of its sine from Table IV. Calculate the area from the formula $\frac{1}{2}AB,\ BC \sin ABC$.

Find the mean of the two answers. State the percentage difference of either from the mean.

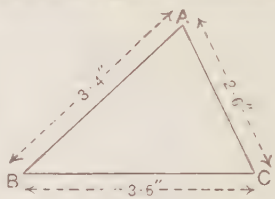


FIG. 125.

22. Draw the quadrilateral $ABCD$, Fig. 126, to the dimensions given. Draw the diagonal AC and measure its length.

Draw perpendiculars on AC from B and D and measure their lengths. From these calculate the area of the quadrilateral.

Draw the other diagonal BD and measure its length, also measure the acute angle between the diagonals (call this θ). Take out $\sin\theta$ from Table IV. and calculate the area of the quadrilateral from the formula $\frac{1}{2}(AC \times BD \sin\theta)$. Find the mean of the two answers and the percentage difference of each from the mean.

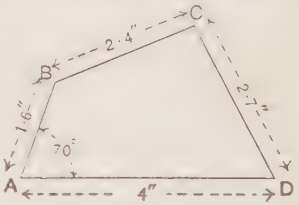


FIG. 126.

MISCELLANEOUS EXERCISES XLVI.

Simplify

1. (*a*) $70 - 16 + 8$; (*b*) $70 - (16 + 8)$.

2. (*a*) $14 + 2 \times 20$; (*b*) $(14 + 2)20$.

3. (*a*) $240 \div 10 - 4$; (*b*) $240 \div (10 - 4)$.

4. Multiply 4 days 8 hrs. 35 min. 20 sec. by 96.

5. Find the difference between 10 metric and 10 English tons. [1 metric ton = 1000 kg., 1 kg. = 2·2 lb.]

6. What is the rate in miles per hour of a motor-car which covers a distance of $\frac{1}{4}$ mile in 26 seconds?

7. A railway train at an average speed of 40 miles per hour takes 27 minutes for a journey. How long would it take at 50 miles an hour?

8. Find the value of (i) $\frac{2}{5}$ of £2. 3*s.* 9$\frac{1}{2}$*d.*, (ii) 0·725 of a guinea.

9. Divide 76544 into two factors of which one is greater than the other by 19132.

10. Find the value of $7\frac{2}{5}$ of £2. 3*s*. $9\frac{1}{2}d$.

11. Add the sum of 45·62, 93·42 and 3·456 to the difference between 83·47 and 59·25.

12. The difference between two numbers is 19132 and the sum is 76544. Find the numbers.

13. Find to the nearest penny the simple interest on £475. 10*s*. for 69 days at $2\frac{1}{2}$ per cent. per annum.

14. A cylindrical can which holds half a gallon is three-quarters full of water. What fraction of a gallon does it contain?

15. A letter in a pair of scales is balanced by 8 pennies and 4 half pennies. If a penny weighs $\frac{1}{3}$ oz. and a half penny $\frac{1}{5}$ oz., what is the weight of the letter?

16. A model of a boat is made to a scale of $\frac{1}{2}$ in. to a foot. If the length of the model is 2 ft., what is the length of the boat?

17. What is the population of a town if $\frac{3}{5}$ of the population amount to 3000?

18. A uniform bar of iron 7 ft. 4 in. long weighs 70 lb. What will be the weight of the remainder when a length 1 ft. 8 in. is cut from it?

19. If 75 yds. of wire weigh 21 lb., what will be the weight of a mile length of the same wire?

20. The time of swing of a pendulum is proportional to the square root of its length. If the time of swing of a pendulum 1 metre long is 1 second, find the length so that the time of swing may be (i) $\frac{1}{4}$ second, (ii) 7 seconds; (iii) find the length so that the pendulum will swing 75 times per minute.

21. The square of the time that a planet takes in its revolution round the sun varies as the cube of its distance from the sun. If the earth at a distance of 93 millions of miles takes $365\frac{1}{4}$ days, what time will be taken by Mercury at a distance of 36 millions of miles?

22. A rod 14 ft. long is to be divided into two parts, one of which is 2·5 times the length of the other; find the length of each.

23. If F denotes the attractive force exerted by a body of mass M_1 on another of mass M_2 then

$$F \propto M_1 M_2 \div r^2 \text{ or } F = k\frac{M_1 M_2}{r^2},$$

where r is the distance between the bodies and k is a constant.

If the constant is G (the Newtonian constant of gravitation) then the mass of the earth can be calculated from the known value of F for a body of mass 1 gram at the surface.

From the formula $F = G\frac{M_1 M_2}{r^2}$ where $F = 981$ dynes,

$G = 0{\cdot}5548 \times 1{\cdot}2 \times 10^{-7}$, $M_1 = 1$ and $r = 3960$ miles, or $6{\cdot}37 \cdot 10^8$ cm.
Calculate M_2 (the mass of the earth) (i) in grams, (ii) in tons.

24. If 90 per cent. of pewter is antimony, what quantity of antimony is there in a pewter vessel weighing 12 oz.?

25. Show that $2\frac{1}{2}$ and 5 per cent. is the same as 6*d.* and 1*s.* in the pound.

Find (i) 5% of £3, £2. 10*s.* and 16*s.* 8*d.*,
(ii) $2\frac{1}{2}$% of £7, £25, £1. 10*s.*, 10*s.* 6*d.*

26. The distance by road between two towns is 30 miles but by railway is $7\frac{1}{2}$% less. Find the distance by the railway.

27. If a pint of water is added to a gallon of pure milk, what is the percentage of milk in the mixture?

28. The population of a town at one census was 26000 and at the next census 30000. What was the percentage increase?
What is the percentage increase in an adjoining town the corresponding populations being 36000 and 40000?

29. What percentage can a debtor pay, who can only pay 12*s.* 6*d.* for each pound he owes?

30. If to a quantity of pure milk 10 per cent. of water is added, what percentage of the mixture is water? What would be the percentage if 100 per cent. of water were added?

31. Half a gallon of wine contains 70 per cent. of pure alcohol. What will be the percentage of alcohol in the mixture if a pint of water is added?

32. The length of a room is increased by 10 per cent. and the width by 5 per cent. What is the percentage increase in the area?

33. A hole in the side of a vessel 1 in. diameter is increased to 2 in. diameter. What is the percentage increase in the area?

34. A rectangular sheet of cardboard 33 cm. long, 22 cm. wide weighs 50 grams. An irregular figure cut out of the cardboard is found to weigh 23 grams. Find the area of the irregular figure.

35. The pressure of wind during a storm was 12·3 lb. per sq. ft. What would be the total pressure on a hoarding 15 yd. long and 12 ft. high?

36. A rectangular plot of land is 80 yd. wide. What is the length if the area is 2 acres? How many yards of fencing will be required to enclose it?

37. Find the number of square feet of timber in a *standard* of deal planks which contains 120 planks, each 12 ft. long and 9 in. wide.

38. What will be the average width of a railway line in which 3 acres of land are used for 1 mile length of the line?

39. A map showing the railways of Canada is drawn to a scale 1 in. = 50 miles. If the map is 7 ft. long and 3 ft. 5 in. wide, what area in square miles is represented?

40. How many meshes are there in a piece of wire netting 6 ft. 6 in. long and 2 ft. 6 in. wide, each mesh being a square of $1\frac{1}{2}$ in. side? What is the length of wire used, allowing 5 per cent. for joints?

41. The sketch (Fig. 127), dimensions in centimetres, shows a gold cross, thickness 2 mm. Find its volume.

42. A steel rule is 2·8 cm. wide, 1 mm. thick, and 31 cm. long. Find its volume and weight.

43. A log of fir, $12\frac{1}{2}$ ft. by 1·25 ft. by 9 in. weighs 387 lb. What is the specific gravity? the weight of the timber per cubic foot?

44. A steel safe is made in the form of a cube, the external length of an edge being 2 ft. 9 in. If the thickness of metal is $1\frac{1}{4}$ in., find the volume and the weight, allowing 10 per cent. for hinges and fastenings.

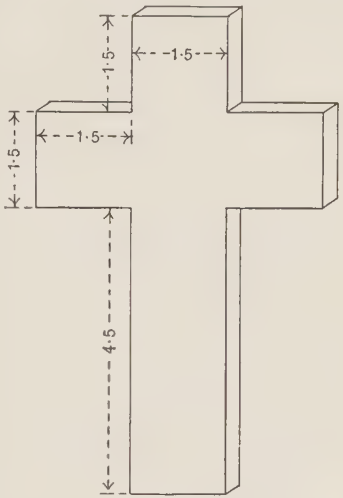


FIG. 127.

45. How many kilograms of water will a rectangular cistern hold, length 2·3 m., width 120 cm., depth 90 cm.?

46. The steel wire for a wire gun is 0·25 in. wide, 0·06 in. thick, and weighs 43·5 lb. Find its length.

47. A swimming bath is 60 ft. long and 20 ft. wide; the depth at one end is 3 ft. and at the other 6 ft. Find the weight of salt water it will contain. [Specific gravity of salt water = 1·026.]

48. A rod of brickwork contains 272 superficial feet (square feet) and is $1\frac{1}{2}$ bricks or $13\frac{1}{2}$ in. thick. Find the volume and weight. [Specific gravity = 1·77.]

49. Find the number of bricks in a rod of brickwork (306 cub. ft.) if each brick occupies a space 9 in. by $4\frac{1}{2}$ in. by 3 in.

50. If the dimensions of a brick are $8\frac{3}{4}$ in. by $4\frac{1}{4}$ in. by $2\frac{3}{4}$ in., find the number of cubic feet of mortar in a rod of brickwork, and its weight. [Specific gravity of mortar = 1·7.]

51. How many tons per acre will a seam of coal 2 ft. thick produce? [Specific gravity = 1·3.]

52. A railway cutting, average depth 15 ft., is 16 ft. wide at the bottom, and the sides slope at an angle of 30° to the horizontal. Find the number of cubic yards of material in a length of a quarter of a mile.

53. A pendulum 1 yard long swings through an angle of 20°. What is the area of the sector moved through in one swing?

54. The minute hand of a clock is 7 ft. long. What distance will its extremity move through in 15 minutes?

55. The section of a bent brass tube (Fig. 128), consists of two straight lengths joined by a semicircle, the external diameter of the tube is 50 mm. thickness 2 mm. Find the volume of water it will hold when full and the total weight of tube and water.

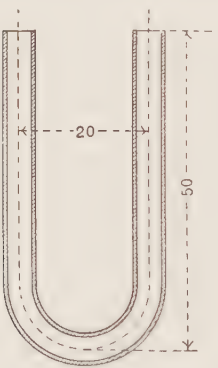


Fig. 128.

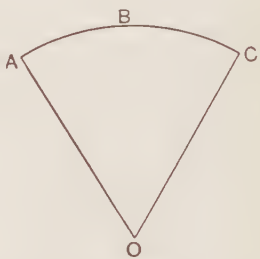


Fig. 129.

56. $OABC$ (Fig. 129) is a segment of a circle (scale $\frac{1}{10}$). Find its area.

57. What weight of mercury will fill a cylindrical glass tube 6 mm. diameter and 1 m. long?

58. 35 grams of mercury are poured into a cylindrical glass tube and occupy a length of 8·5 cm. Find the diameter of the tube.

59. The cross-section of a graduated glass cylinder is given in Fig. 130. Find the vertical distance between the marks indicating 100 c.c. and 1000 c.c.

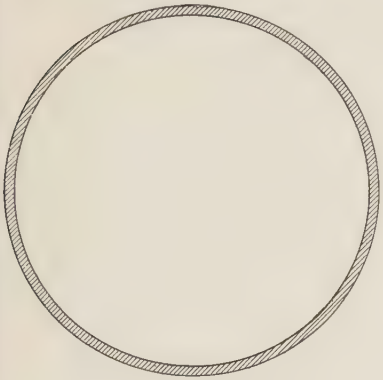


FIG. 130.—Scale $\frac{1}{2}$ full size.

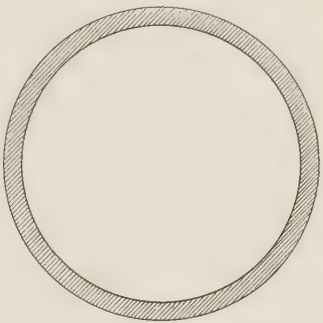


FIG. 131.—Scale $\frac{1}{4}$ full size.

60. In Fig. 131 the cross-section of a cast-iron pipe is given. Find the weight of the pipe in pounds per lineal foot.

61. The section of a glass flask, scale half full size, is shown in Fig. 132 and consists of a cylinder and a sphere. Find approximately how many cubic centimetres of water it will hold when full.

62. Find the weight that a spherical balloon 45 ft. diameter will lift when filled with coal gas. [Specific gravity = 0·44 (that of air being taken as unity). 1 cub. ft. of air weighs 0·0807 lb.]

The lifting force is the difference between the weight of the gas it contains and that of the same volume of air.

63. A sphere 10 in. radius is coated with lead to a uniform thickness of $\frac{1}{10}$ in. Find the volume of the lead and show that the volume is approximately equal to the surface of the sphere multiplied by the thickness of the lead.

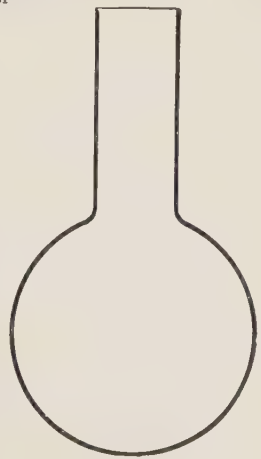


FIG. 132.

64. When a sphere is immersed completely it is found to displace 7 gallons of water. If the internal diameter of the sphere is 14·47 in., what is its thickness? [1 gall. = 0·1604 cub. ft.]

65. A rope is coiled round a rough post. If P is the pull at one end and T that at the other, then $P = Te^{\mu\theta}$.
Given $\mu = 0{\cdot}3$, $\theta = 6{\cdot}28$, $T = 100$ lb., $e = 2{\cdot}718$, find P.

CHAPTER XI.

ARITHMETICAL EVALUATION.

Arithmetical Evaluation.—Many examples of arithmetical evaluation are to be found in the preceding pages, for example, in the sections dealing with length, area and volume; but more difficult examples than those already dealt with are met with continually in practical work. These evaluations are so numerous and so varied, that it is very difficult to give a typical selection; it is, in fact, only possible to consider a few of the more commonly occurring cases.

Rectangular Beam.—The strength of a bar of wood, or other material, when supported at its ends and loaded at some intermediate point, is proportional to its breadth and the square of its depth. These facts may be written in the form

$$S \propto b \times d^2; \quad \therefore \ S = kbd^2,$$

where S denotes the strength of a beam to resist cross-breaking, b is its breadth, d its depth and k is a constant.

The value of k—which may be obtained by experiment—is tabulated for various materials. Hence, the strength of a beam of any given material may be obtained by a simple arithmetical operation.

EXAMPLE 1. *Find the ratio of the strengths of a rectangular wooden beam* (i) *when placed so that the larger dimension AB* (Fig. 133) *is in a horizontal direction*; (ii) *when the smaller dimension BC is horizontal.*

The depth in (i) will be 3 in. and in (ii) will be 11 in.

(i) $S = k \times 11 \times 3^2 = k \times 11 \times 9,$

(ii) $S_1 = k \times 3 \times 11^2 = k \times 3 \times 121;$

$$\therefore \text{ ratio} = \frac{S_1}{S} = \frac{3 \times 121}{11 \times 9} = \frac{11}{3}.$$

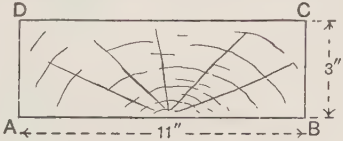


FIG. 133.

So that the beam is nearly four times as strong when placed with its greatest dimension in a vertical direction than in the other direction.

EXAMPLE 2. *The moment of inertia of a thin rod about a line passing through its middle point and at right angles to the axis of the rod is given by*

$$I = \frac{W}{g} \times \frac{l^2}{12}.$$

Find the numerical value of I, given $W = 4$, $l = 25$, $g = 32{\cdot}2$.

Substituting the given values,

$$I = \frac{4}{32{\cdot}2} \cdot \frac{25^2}{12} = \frac{25^2}{96{\cdot}6} = 6{\cdot}469.$$

$\log 25 = 1{\cdot}3979$
$2 \log 25 = 2{\cdot}7958$
$\log 96{\cdot}6 = 1{\cdot}9850$ antilog $0{\cdot}8108 = 6{\cdot}469.$
$0{\cdot}8108$

EXAMPLE 3. *The moment of inertia of a circle about a line in the plane of the figure and passing through the centre of the circle is given by* $I = \frac{\pi}{64} \times d^4$, *where d is the diameter of the circle. Given* $\pi = 3{\cdot}142$, $d = 2{\cdot}5$ *in., find the numerical value of I.*

Substituting the given values,

$$I = \frac{3{\cdot}142}{64} \times (2{\cdot}5)^4 = 1{\cdot}917.$$

$4 \log 2{\cdot}5 = 1{\cdot}5916$
$\log 3{\cdot}142 = 0{\cdot}4972$
$2{\cdot}0888$
$\log 64 = 1{\cdot}8062$ antilog $0{\cdot}2826 = 1{\cdot}917.$
$0{\cdot}2826$

EXAMPLE 4. *The dimensions of the cotter for a shaft of diameter D may be found from* $b = 1{\cdot}25D$, $t = 0{\cdot}261D$. (i) *Given* $D = 4$ *in., find b and t. If* $d = D \div 1{\cdot}23$, *find d.*

(i) $b = 1{\cdot}25 \times 4 = 5$ in.,

$t = 0{\cdot}261 \times 4 = 1{\cdot}044$ in.

$d = 4 \div 1{\cdot}23 = 3{\cdot}25$ in.

EXAMPLE 5. *The brake horse power P of an engine may be found from*

$$P = \frac{2 \times \pi \times R \times N}{33000}(W - w).$$

Given $\pi = 3{\cdot}142$, $R = 6{\cdot}5$, $N = 80$, $W = 250$ *and* $w = 75$, *find P.*

Substituting these values,

$$P=\frac{2\times 3\cdot 142\times 6\cdot 5\times 80}{33000}(250-75);$$

$$\therefore\ P=\frac{2\times 3\cdot 142\times 6\cdot 5\times 80\times 175}{33000}$$

$$=17\cdot 33.$$

$\log 2=0\cdot 3010$
$\log 3\cdot 142=0\cdot 4972$
$\log 6\cdot 5=0\cdot 8129$
$\log 80=1\cdot 9031$
$\log 175=2\cdot 2430$
$5\cdot 7572$
$\log 33000=4\cdot 5185$ antilog $1\cdot 2387=17\cdot 33$.
$1\cdot 2387$

Example 6. *The length (in feet) of a pendulum is given by* $l=gt^2\div\pi^2$. *Find* l *when* $g=32\cdot 2$, $\pi=3\cdot 142$ *and* $t=1$.

$$l=\frac{32\cdot 2}{(3\cdot 142)^2}=3\cdot 262 \text{ ft.}$$

$\log 32\cdot 2=1\cdot 5079$
$2\log 3\cdot 142=0\cdot 9944$ antilog $0\cdot 5135=3\cdot 262$ ft.
$0\cdot 5135$

Example 7. *The diameter of a rivet may be obtained from* $d=1\cdot 2\sqrt{t}$, *where* d *denotes the diameter of the rivet and* t *the thickness of the plate. Find values of* d (i) *when* t *is* $\frac{3}{8}$ *in.*, (ii) *when* t *is* $\frac{5}{8}$ *in.*

(i) $\frac{3}{8}=0\cdot 375$;

$$\therefore\ d=1\cdot 2\sqrt{0\cdot 375}=0\cdot 7348 \text{ in.}$$

$\log 0\cdot 375=\bar{1}\cdot 5740$
$\frac{1}{2}\log 0\cdot 375=\bar{1}\cdot 7870$ antilog $\bar{1}\cdot 8662=0\cdot 7348$.
$\log 1\cdot 2=0\cdot 0792$
$\bar{1}\cdot 8662$

(ii) $\frac{5}{8}$ in. $=0\cdot 625$ in.;

$$d=1\cdot 2\sqrt{0\cdot 625}=0\cdot 9486 \text{ in.}$$

$\frac{1}{2}\log 0\cdot 625=\bar{1}\cdot 8979$
$\log 1\cdot 2=0\cdot 0792$ antilog $\bar{1}\cdot 9771=0\cdot 9486$.
$\bar{1}\cdot 9771$

EXAMPLE 8. *The thickness of the shell of a boiler may be found from*

$$t=\frac{p\times d}{2\times f\times e}.$$

Given $p=125$ *lb. per sq. in.,* $d=90$ *in.,* $f=1{\cdot}34\times10^4$ *lb. per sq. in. and* $e=0{\cdot}7$, *find* t.

Substituting these values,

$$t=\frac{125\times90}{2\times1{\cdot}34\times10^4\times0{\cdot}7}=0{\cdot}5996 \text{ in.}$$

$\log 125=2{\cdot}0969$
$\log 90=1{\cdot}9542$
$4{\cdot}0511$
$4{\cdot}2732$
$\overline{1}{\cdot}7779$

$\log 2=0{\cdot}3010$
$\log 1{\cdot}34\times10^4=4{\cdot}1271$
$\log 0{\cdot}7=\overline{1}{\cdot}8451$
$4{\cdot}2732$

$$\text{antilog}\,\overline{1}{\cdot}7779=0{\cdot}5996.$$

EXAMPLE 9. *The twisting moment* T *that can be transmitted by a shaft of diameter* d *is given by*

$$T=\frac{\pi}{16}fd^3.$$

If $f=9\times10^3$, $d=3\frac{1}{4}$, *find* T.

Substituting these values,

$$T=\frac{3{\cdot}142\times9\times10^3\times(3{\cdot}25)^3}{16}=60670.$$

$\log 3{\cdot}142=0{\cdot}4972$
$\log 9000=3{\cdot}9542$
$3\log 3{\cdot}25=1{\cdot}5357$
$5{\cdot}9871$
$\log 16=1{\cdot}2041$
$4{\cdot}7830$

$\text{antilog}\,4{\cdot}7830=60670.$

EXAMPLE 10. *The horse power absorbed in friction in a footstep bearing when the pressure is* p, *the number of revolutions* n, *and the radius* r *is given by*

$$P=\frac{4\times\mu\times p\times\pi\times r\times n}{3\times33000}.$$

If $p=3000$ *lb.,* $n=100$, $\mu=0{\cdot}06$, $r=\frac{1}{6}$ *ft., find* P.

Substituting these values,

$$P=\frac{4\times 0{\cdot}06\times 3000\times 3{\cdot}142\times 100}{3\times 33000\times 6}$$

$$=\frac{4\times 0{\cdot}06\times 314{\cdot}2}{198}=0{\cdot}38.$$

$\log 4 = 0{\cdot}6021$
$\log 0{\cdot}06 = \bar{2}{\cdot}7782$
$\log 314{\cdot}2 = 2{\cdot}4972$
$1{\cdot}8775$
$\log 198 = 2{\cdot}2967$
$1{\cdot}5808$

antilog $\bar{1}{\cdot}5808 = 0{\cdot}3809.$

Example 11. *The torsional strength of a solid shaft of diameter d equivalent to that of a solid shaft of the same material, diameters D_2 and D_1, may be found from*

$$d^3=\frac{D_2^4-D_1^4}{D_2}.$$

Given $D_2=14$ in., $D_1=10$ in., find d.

Substituting, $$d^3=\frac{14^4-10^4}{14}=\frac{28416}{14}=\frac{7104\times 4}{14};$$

$$\therefore\ d=12{\cdot}66 \text{ in.}$$

$\log 7104 = 3{\cdot}8515$
$\log 4 = 0{\cdot}6021$
$4{\cdot}4536$
$1{\cdot}1461$
$3 \mid 3{\cdot}3075$
$1{\cdot}1025$

antilog $1{\cdot}1025 = 12{\cdot}66.$

Example 12. *When a shaft is subjected to combined twisting and bending moment, the equivalent twisting moment T_e is given by*

$$T_e=M+\sqrt{M^2+T^2},$$

where M denotes the bending and T the twisting moment on the shaft. Given $M=40$, $T=30$, find T_e.

Substituting the given values,

$$T_e=40+\sqrt{40^2+30^2}$$

$$=40+\sqrt{2500}=90.$$

EXAMPLE 13. *At the centre of a thin coil of a torus carrying a current C, the intensity of a magnetic field is given by*

$$F = 2 \times \pi \times n \times C \div r.$$

Find the value of F when $\pi = 3{\cdot}142$, $n = 250$, $C = 1{\cdot}6$, $r = 2{\cdot}4$.

Substituting the given values,

$$F = \frac{2 \times 3{\cdot}142 \times 250 \times 1{\cdot}6}{2{\cdot}4} = 1047.$$

EXAMPLE 14. *The horse power P that can be transmitted by a shaft of diameter d when the twisting moment is T may be found from*

$$P = \frac{2 \times \pi \times n \times T}{33000}.$$

Given $\pi = 3{\cdot}142$, $n = 250$, $T = 20000$, *find P.*

Substituting, $$P = \frac{2 \times 3{\cdot}142 \times 250 \times 20000}{33000} = 951{\cdot}9.$$

$\log 2 = 0{\cdot}3010$
$\log 3{\cdot}142 = 0{\cdot}4972$
$\log 250 = 2{\cdot}3979$
$\log 20 = 1{\cdot}3010$
$4{\cdot}4971$
$\log 33 = 1{\cdot}5185$ antilog $2{\cdot}9786 = 951{\cdot}9$.
$2{\cdot}9786$

EXAMPLE 15. *If there are n cells in series, each of resistance r, the current C is given by*

$$C = \frac{n \times E}{R + nr},$$

where E denotes the E.M.F. *and R the external resistance. Given* $r = 2{\cdot}6$, $R = 100$, $E = 1{\cdot}8$, *find C when there are 5 cells in series.*

Substituting the given values, we obtain

$$C = \frac{5 \times 1{\cdot}8}{100 + 5 \times 2{\cdot}6} = \frac{9}{113} = 0{\cdot}0796.$$

EXAMPLE 16. *The power given to an external circuit by means of a battery of internal resistance r, external resistance R and* E.M.F. *denoted by E is given by*

$$W=\frac{R\times E^2}{(R+r)^2}.$$

Given $R=1\cdot6$, $E=3$, *and* $r=1\cdot5$, *find* W.

Substituting the given values,

$$W=\frac{1\cdot6\times3^2}{(1\cdot6+1\cdot5)^2}=\frac{14\cdot4}{(3\cdot1)^2},$$

$\log 14\cdot4 = 1\cdot1584$
$2\log 3\cdot1 = 0\cdot9828$ antilog $0\cdot1756 = 1\cdot498$.
$0\cdot1756$

EXAMPLE 17. *The electrical resistance of a wire varies directly as its length and inversely as the area of its cross-section. If the resistance of* 1000 *yards of wire* 4·465 *mm. diameter is* 1·02 *ohms, what is the resistance of a mile of wire of the same material, diameter* 3·251 *mm.*

Let R denote the resistance, l the length and A the sectional area of the wire.

Then $R=\frac{k\times l}{A}$ where k is a constant.

Substituting the given values for the first wire, we obtain

$$1\cdot02=\frac{k\times1000}{A},$$

where $A=0\cdot7854\times d^2=0\cdot7854\times(4\cdot465)^2=15\cdot66$;

$$\therefore\ k=\frac{15\cdot66\times1\cdot02}{1000}.$$

In the second case $R=\frac{k\times l}{A}$,

where $A=0\cdot7854\times(3\cdot251)^2=8\cdot301$;

$$\therefore\ R=\frac{15\cdot66\times1\cdot02}{1000}\times\frac{1760}{8\cdot301}=3\cdot386 \text{ ohms}.$$

$\log 15\cdot66 = 1\cdot1947$
$\log 1\cdot02 = 0\cdot0086$
$\log 1760 = 3\cdot2455$ antilog $0\cdot5296 = 3\cdot386$
$4\cdot4488$
$\log 8301 = 3\cdot9192$
$0\cdot5296$

EXAMPLE 18. *A beam supported at each end, of length l, breadth b and depth d, will safely carry a load W at the centre of the beam when*

$$W = \frac{8 \times I \times f}{l \times L} \quad \text{.....(i)}$$

I for a rectangular section $= \frac{1}{12} bd^3$ and for a circular section $= \frac{\pi}{64} D^4$.

Given $b = 1{\cdot}2$ in., $d = 1{\cdot}5$ in., $l = 72$ in., $f = 10^4$ lb. per sq. in., find W.

Substituting the given values,

$$W = \frac{8 \times 1{\cdot}2 \times 1{\cdot}5^3 \times 10^4}{72 \times 12 \times 1{\cdot}5} = \frac{8 \times 12000 \times 1{\cdot}5^2}{72 \times 12}$$

$$= \frac{8000 \times 1{\cdot}5^2}{72} = 250 \text{ lb.}$$

$\log 8000 = 3{\cdot}9031$
$2 \log 1{\cdot}5 = 0{\cdot}3522$
$4{\cdot}2553$
$1{\cdot}8573$
$2{\cdot}3980$

$\text{antilog } 2{\cdot}3980 = 250.$

EXAMPLE 19. *The deflection, D in inches, of the beam in Example 18 when loaded at the centre of the beam with a load W is given by*

$$D = \frac{Wl^3}{48EI} \quad \text{.....(ii)}$$

Find the deflection, $E = 3 \times 10^7$ lb. per sq. in.

Substituting the given values,

$$D = \frac{250 \times 72^3 \times 12}{48 \times 3 \times 10^7 \times 1{\cdot}2 \times 1{\cdot}5^3} = 0{\cdot}192 \text{ in.}$$

$\log 250 = 2{\cdot}3979$
$3 \log 72 = 5{\cdot}5719$
$\log 12 = 1{\cdot}0792$
$9{\cdot}0490$
$9{\cdot}7658$
$1{\cdot}2832$

$\log 48 = 1{\cdot}6812$
$\log 3 \times 10^7 = 7{\cdot}4771$
$\log 1{\cdot}2 = 0{\cdot}0792$
$3 \log 1{\cdot}5 = 0{\cdot}5283$
$9{\cdot}7658$

$\text{antilog } 1{\cdot}2832 = 0{\cdot}192 \text{ in.}$

EXAMPLE 20. *When the beam is circular in cross-section find the safe load W which may be applied at the centre of the beam and the deflection produced by the load when $d = 2{\cdot}5$ in., where d is the diameter of cross-section, the length l being 72 in., $f = 10^4$ lb. per sq. in., $E = 3 \times 10^7$ lb. per sq. in.*

The numerical value of W is obtained from

$$W = \frac{8 \times I \times f}{l \times d},$$

where

$$I = \frac{\pi}{64} \times (2{\cdot}5)^4.$$

Substituting these values,

$$W = \frac{8 \times \pi \times 2{\cdot}5^4 \times 10^4}{64 \times 72 \times 2{\cdot}5} = \frac{8 \times \pi \times 2{\cdot}5^3 \times 10^4}{64 \times 72} = 852{\cdot}1 \text{ lb.}$$

$3 \log 2{\cdot}5 = 1{\cdot}1937$
$\log \pi = 0{\cdot}4972$
$\log 8 = 0{\cdot}9031$
$4 \log 10 = 4{\cdot}0000$
$6{\cdot}5940$
$3{\cdot}6635$
$2{\cdot}9305$

$\log 64 = 1{\cdot}8062$
$\log 72 = 1{\cdot}8573$
$3{\cdot}6635$

$$\text{antilog } 2{\cdot}9305 = 852{\cdot}1 \text{ lb.}$$

To obtain the deflection. Substitute the given values in (ii).

$$D = \frac{852{\cdot}1 \times 72^3 \times 64}{48 \times 3 \times 10^7 \times \pi \times (2{\cdot}5)^4} = 0{\cdot}1152 \text{ in.}$$

$\log 852{\cdot}1 = 2{\cdot}9305$
$3 \log 72 = 5{\cdot}5719$
$\log 64 = 1{\cdot}8062$
$10{\cdot}3086$
$11{\cdot}2471$
$\bar{1}{\cdot}0615$

$4 \log 2{\cdot}5 = 1{\cdot}5916$
$\log 3{\cdot}142 = 0{\cdot}4972$
$\log 3 \times 10^7 = 7{\cdot}4771$
$\log 48 = 1{\cdot}6812$
$11{\cdot}2471$

$$\text{antilog } \bar{1}{\cdot}0615 = 0{\cdot}1152 \text{ in.}$$

EXERCISES XLVII.

1. The width W across the flats of a hexagonal nut is given by $W=1\frac{1}{2}d+\frac{1}{8}''$. Find W when $d=1''$, $1\frac{1}{2}''$, $2''$.

2. The pitch p of a riveted joint may be obtained from

$$(p-d)t=0{\cdot}7854d^2.$$

Given $t=\frac{3}{8}''$, $d=\frac{3}{4}''$, find p.

3. The space S described by a body moving with an initial velocity V and acceleration g for a time t is given by

$$S=Vt+\tfrac{1}{2}gt^2.$$

Given $V=50$, $t=3$, $g=32{\cdot}2$, find S.

4. The space S from rest of a body moving with uniform acceleration g for a time t may be found from

$$S=\tfrac{1}{2}gt^2.$$

Find t when $S=200$, $g=32{\cdot}2$.

5. The moment of inertia of a rod may be found from

$$I=\frac{W}{g}\frac{l^2}{12}.$$

Find I when $W=5$, $l=35$, $g=32{\cdot}2$.

6. The moment of inertia of a circle of diameter d is given by

$$I=\frac{\pi}{64}d^4.$$

Find I (i) when $d=\frac{5}{8}''$, (ii) when $d=1{\cdot}25''$.

7. The dimensions b and t of a cotter may be obtained from

$$b=1{\cdot}25D,\quad t=0{\cdot}261D,\quad D=1{\cdot}2d.$$

Given $d=5''$, find b and t.

8. The brake horse power of an engine is given by

$$P=\frac{2\pi RN(W-w)}{33000}.$$

Find P when $R=9{\cdot}5$, $N=90$, $W=320$, $w=60$.

9. The length l of a pendulum may be found from

$$l=gt^2\div\pi^2.$$

Find l (i) when $t=1{\cdot}5$, (ii) when $t=3$ and $g=32{\cdot}2$.

10. The thickness t of the shell of a boiler may be obtained from

$$t=\frac{pd}{2fe}.$$

(i) Given $p=100$ lb. per sq. in., $d=87''$, $f=10000$, and $e=0{\cdot}56$, find t.

(ii) If $t=\frac{5}{8}$ in., $d=87$ in., $f=10000$, and $e=0{\cdot}56$, find p.

11. The volume of the frustum of a pyramid, height h, area of base A_1, area of parallel face A_2, is given by

$$V=\frac{h}{3}(A_1+A_2+\sqrt{A_1A_2}).$$

Given $h=10$, $A_1=36$ and $A_2=16$, find V.

12. The volume of the frustum of a cone, height h, radius of base r_1, radius of parallel face r_2, is given by

$$V=\frac{\pi h}{3}(r_1^2+r_2^2+r_1r_2).$$

Find the numerical value of V when $h=25$, $r_1=10$, $r_2=5$ and $\pi=3{\cdot}142$.

13. The volume of a zone of a sphere may be found from

$$V=\frac{\pi h}{6}(3R^2+3r^2+h^2).$$

Given $R=6$, $r=5$, $h=2$, $\pi=3{\cdot}142$, find the numerical value of V.

14. The work lost by friction in a cylindrical journal of diameter d may be found from

$$W=\frac{\pi\times\mu\times P\times d\times n}{12}.$$

Given $\mu=0{\cdot}05$, $P=1000$ lb., $d=4$ in. and $n=100$, find W.

15. The diameter of a rivet may be obtained from $d=1{\cdot}2\sqrt{t}$, where d denotes the diameter of the rivet, and t the thickness of the plates. Find the diameters of rivets when the thickness of plate is (i) $\frac{1}{2}$ in., (ii) $\frac{3}{4}$ in., (iii) $\frac{7}{8}$ in.

16. Given

$$v=\sqrt{\frac{g\times f}{450}},$$

if $f=3000$, $g=32{\cdot}2$, find v.

17. Given $d=0{\cdot}038\sqrt{(D\times l\times\sqrt{p})}$, if $D=9{\cdot}35$, $p=280$ and $l=84$, find d.

18. If $D=0{\cdot}528\times 30^{0{\cdot}57}$, find D.

19. The diameter of the bolts of a flange coupling is given by

$$d=0{\cdot}577\sqrt{\frac{D^3}{N\times r}},$$

where D is the diameter of the shaft, N the number of bolts and r the radius of the bolt circle. Find the diameter of each of the six bolts of such a coupling, the bolt circle being 18 in. diameter, and the shaft $12\frac{1}{4}$ in. diameter.

20. The horse power transmitted by a belt of width w, thickness t, and velocity v ft. per sec. is given by

$$P=\frac{(24150-v^2)b\times t\times v}{88550}.$$

Find P, given $v=90$, $b=2{\cdot}7$ and $t=\frac{3}{8}$.

21. The loss of "head" (due to friction, etc. of water in a cast-iron pipe when the velocity is v ft. per sec., l = length in feet, and diameter d ft., may be found from

$$h=\frac{0{\cdot}0243}{d^{1{\cdot}168}}\times l\times\frac{v^2}{2g}.$$

Find h when $l=1200$, $v=4$, $d=1\frac{1}{2}$ ft. and $g=32{\cdot}2$.

22. The deflection or droop of a rectangular beam supported at each end and loaded uniformly is given by

$$D=\frac{5}{384}\times\frac{W\times l^3\times 12}{E\times b\times d^3}.$$

Given $W=2400$ lb., $l=240$ in., $E=1444000$ lb. per sq. in., $b=4$ in., $d=11{\cdot}4$ in., find D.

23. In a beam supported at each end the load W that may be safely carried at the centre of the beam is given by $W=\frac{8\times I\times f}{l\times d}$. Find W when $b=1\frac{1}{8}$ in., $d=2\frac{3}{4}$ in., $I=b\times d^3\div 12$, $f=10^4$, $l=72$ in. Also find from

$$D=Wl^3\div(48\times E\times I)$$

the deflection produced by this load ($E=3\times 10^7$).

24. The tensions T and t at the two ends of a cord wrapped round a post are given by $T=t\times 2{\cdot}718^{\mu\theta}$. Find T when $t=50$, $\mu=0{\cdot}35$ and $\theta=2\pi$.

25. A Daniell's cell has an E.M.F. of 1·07 volts and an internal resistance of 10 ohms. What current will it send through an external resistance of 32 ohms (p. 214)?

26. A battery having an E.M.F. of 15 volts, and an internal resistance of 25 ohms, is used to send a current through a wire having a resistance of 50 ohms. What is the potential difference at the battery terminals?

27. What current must the battery (in Question **26**) send so that the terminal potential difference at the battery may be 7·5 volts?

28. If a battery having an E.M.F. of 8 volts has its terminal potential difference reduced to 2 volts on sending a current of 2 amperes, what is its internal resistance?

29. A battery has a terminal potential difference of 12 volts when sending a current of 3 amperes, what is its internal resistance?

30. 50 Grove cells (E.M.F. = 1·8 volts) are united in series and the circuit is completed by a wire whose resistance is 15 ohms. Find the strength of the current.

31. The current through the carbon filament of an incandescent lamp was found to be 1 ampere. The difference of potential between the two terminals of the lamp whilst the current was passing, was found to be 30 volts. What was the resistance of the filament?

32. Calculate the number of cells required to produce a current of 50 milliamperes (0·05 amp.) through a line 114 miles long, whose resistance is $12\frac{1}{2}$ ohms per mile, the available cells of the battery having each an internal resistance of 1·5 ohm and an E.M.F. of 1·5 volts.

33. A battery has a terminal potential difference of 15 volts when sending a current of 2 amperes. Find the internal resistance of the battery.

34. A ten-cell Daniell battery (E.M.F. 1·07 volts), resistance 10 ohms per cell, is used to work a telegraph line 10 miles long, the resistance of the wire being 14 ohms per mile. If the resistance of the instrument is 220 ohms, determine the strength of the current.

35. A generator giving a constant potential difference of 220 volts sends a current of 90 amperes when a certain external resistance is in circuit. Find the resistance.

36. An electric motor is supplied with current from constant potential mains giving 220 volts. If the motor takes 15 amperes, and the resistance of the motor and connecting leads be 5 ohms, determine the back E.M.F. of the motor.

37. The section of a water course is given (Fig. 134). If the surface velocity of the water is 4 miles per hour, and mean velocity = (surface velocity) × 0·8, find the discharge in gallons per minute when the height H of the water is (i) 2 ft., (ii) 4 ft., (iii) 5 ft.

FIG. 134.

38. If an object h ft. above the sea level is just visible above the sea horizon to a person whose eye is H feet above the sea level, the distance of the object from the observer is $\frac{2}{3}(\sqrt{h} + \sqrt{H})$ miles.

(i) What is the distance of a steamer when the hull, 12 ft. high, is just visible to a person whose eye is 10 ft. above the sea?

(ii) Find the rate at which a steamer is moving directly from a person, whose eye is 10 ft. above the sea, it being noted that the top of the mast, 50 ft. high, disappears 15 minutes after the hull, the top of which is 12 ft. above sea level.

(iii) Find the length of time after leaving land before the steamer disappears.

39. The quantity of water Q cub. ft. flowing over a rectangular notch per second is given by $Q = 3{\cdot}33\left(L - \frac{H}{5}\right)H^{\frac{3}{2}}$. If L is 2 ft., find Q when H is (i) 2 ft., (ii) 4 ft.

40. The safe load W at the centre of a beam of length l, breadth b and depth d supported at each end is obtained from

$$W = \frac{8If}{ld}.$$

Given $b = 1{\cdot}5''$, $d = 1\frac{3}{4}''$, $f = 10^4$, $l = 60''$ and $I = \frac{bd^3}{12}$, find W.

41. The droop D of a beam supported at each end and loaded at the centre is given by $Wl^3 \div 48EI$. Find D for the beam in Ex. **40**, given

$$E = 3 \times 10^7.$$

42. The safe load at the centre of a beam of diameter d supported at each end and loaded with a weight W at the centre is given by

$$W = 8If \div ld.$$

The droop D at the middle caused by the weight W is given by

$$Wl^3 \div 48EI.$$

Given $d = 3{\cdot}5''$, $I = \frac{\pi}{64}d^4$, $f = 10^4$, $l = 60''$ and $E = 3 \times 10^7$, find W and D.

43. A cistern containing water has an orifice in one of its sides of area a sq. ft. If h denotes the height of the surface of the water above the centre of the orifice, Q the quantity of the water in cubic feet flowing per sec., then $Q = ac\sqrt{2gh}$, where $g = 32{\cdot}2$, $c = 0{\cdot}6$.

Find the number of gallons G when h is constant and equal to 2 ft.

(i) When the orifice is in the form of an equilateral triangle of 2 in. side.

(ii) A circle, 2 in. diameter.

(iii) A rectangle, 2 in. by 4 in.

(iv) A square, 3″ side. $[G = Q \div 0{\cdot}1604.]$

44. If Q denotes the quantity of water in cubic feet per sec. flowing over a V-notch (Fig. 135) then

$$Q = 2{\cdot}635H^{\frac{5}{2}},$$

where H is the height in feet. Find the number of gallons passing per second when H is (i) 2 ft., (ii) 4 ft.

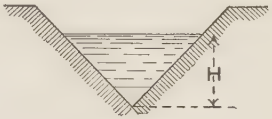


FIG. 135.

45. The twisting moment T that can be transmitted by a shaft of diameter d is given by

$$T = \frac{\pi}{16}fd^3, \text{ or by } \frac{\pi}{16}f\left(\frac{D^4 - d^4}{D}\right),$$

where D is the external and d the internal diameter of a hollow shaft.

(i) Given $f = 9000$ and $d = 2{\cdot}5''$, find T.

(ii) Given $f = 9000$ and $D = 12''$, $d = 6''$, find T.

46. The horse power P transmitted by a shaft of diameter d when the twisting moment is T is given by

$$P = \frac{2\pi NT}{33000 \times 12}.$$

Given $N = 100$ and $T = 27610$ in lb., find P.

TABLES

MONEY.

4 farthings	= 1 penny (*d.*).
12 pence	= 1 shilling (*s.*).
20 shillings	= 1 pound (£) or 1 sovereign.

MEASURES OF LENGTH.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
5½ yards	= 1 rod, pole, or perch.
22 yards	= 1 chain.
1760 yards	= 1 mile.

The **chain** used in surveying consists of 100 links.

1 nautical mile	= 6086 feet.
1 fathom	= 6 feet.

MEASURES OF AREA.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
10,000 square links	= 1 square chain.
10 square chains	= 100,000 square links = 1 acre = 4840 square yards.
640 acres	= 1 square mile.

MEASURES OF VOLUME.

1728 cubic inches	= 1 cubic foot (cub. ft.).
27 cubic feet	= 1 cubic yard (cub. yd.).

MEASURES OF CAPACITY.

4 gills = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gall.).

AVOIRDUPOIS WEIGHT.

16 drams (drs.) = 1 ounce (oz.).
16 ounces (7000 grains) = 1 pound (lb.).
14 pounds = 1 stone (st.).
28 pounds = 1 quarter (qr.).
4 quarters (112 lb.) = 1 hundredweight (cwt.).
20 cwts. (2240 lb.) = 1 ton.

TROY WEIGHT.

(Used for gold and silver.)

24 grains = 1 pennyweight (dwt.).
20 dwt. = 480 grains = 1 ounce (oz. tr.).
12 ounces (5760 grains) = 1 pound (lb. tr.).

TIME.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (d.).
7 days = 1 week (w.).
365 days = 1 year (y.).
366 days = 1 leap year.

METRIC TABLES.

MEASURES OF LENGTH.

10 millimetres (mm.) = 1 **centimetre** (cm.)
10 centimetres = 1 decimetre (dcm.).
10 decimetres = 1 **metre** (m.).
10 metres = 1 decametre.
10 decametres = 1 hectometre.
10 hectometres = 1 **kilometre** (km.).

MEASURES OF AREA OR SQUARE MEASURE.

100 square millimetres = 1 square centimetre (sq. cm.).
100 square centimetres = 1 square decimetre (sq. dcm.).
100 square decimetres = 1 square metre.
1 hectare = 100 ares = 10,000 sq. m.

CUBIC MEASURE.

1000 cubic millimetres = 1 **cubic centimetre** (c.c.).
1000 cubic centimetres = 1 cubic decimetre = 1 **litre.**
1000 cubic decimetres = 1 **cubic metre.**

10 milligrams = 1 centigram.
10 centigrams = 1 decigram.
10 decigrams = 1 **gram** (gm.).
1000 grams = 1 **kilogram** (kilog.).

A gram is the weight of 1 c.c. of water at a temperature 4° C.

CONVERSION TABLE.

MEASURES OF LENGTH.

1 millimetre (mm.) = 0·001 m. = 0·0394 inch.
1 centimetre (cm.) = 0·01 m. = 0·394 inch.
1 decimetre (dcm.) = 3·937 inches.
1 metre (m.) 100 cm. = { 39·37 in., 3·281 ft., 1·094 yds., 0·001 km.
1 kilometre (km.) = 0·621 mile.

1 inch = 2·54 centimetres.
1 foot = 12 in. = 30·48 cm. = 0·3048 m.
1 yard = 3 ft. = 0·9144 metre (m.).
1 mile = 1760 yds. = 5280 ft. = 1609·3 m. = 1·6093 km.

MEASURES OF AREA OR SQUARE MEASURE.

1 square centimetre (sq. cm.) = 0·155 square inch (sq. in.).
1 square decimetre (100 sq. cm.) = 15·5 square inches.
1 square metre (m.) = 10·764 sq. ft. = 1·196 sq. yd.
1 square kilometre (km.) = 0·3861 square mile.

1 square inch (sq. in.) = 6·451 square centimetres (sq. cm.).
1 square foot or 144 sq. in. = 929 square centimetres.
1 square yard (sq. yd) or 9 sq. ft. = 8361 sq. cm. = 0·836 sq. m.
1 acre = 4840 sq. yd. = 4046·7 sq. m. = 0·4047 hectares.
1 square mile = 2·55 square kilometres.

CUBIC MEASURE.

1 cubic centimetre (c.c.) = 0·061 cubic inch.
1 cubic decimetre (c.dcm.) = 1000 c.c. = 61·03 cub. in.
1 cubic metre (1000 c.dcm.) = 35·31 cub. ft. = 1·31 cub. yd.

1 cubic inch (cub. in.) = 16·387 cubic centimetres (c.c.).
1 cubic foot (1728 cub. in.) = 28316 c.c. = 0·283 cub. m.
1 cubic yard (27 cub. ft.) = 764535 c.c. = 0·7645 cub. m.

MEASURES OF CAPACITY.

1 centilitre (0·01 litre) = 0·07 gill.
1 decilitre (0·1 litre) = 0·176 pint.
1 **litre** = 1·76 pints = 0·22 gallon.

1 gill = 1·42 decilitres.
1 pint = 0·568 litre.
1 quart = 1·136 litres.
1 gallon (0·1604 cub. ft.) = 4·546 litres.

1 gram = 15·432 grains.
1 **kilogram** = 1000 grams = 2·2 lb. = 0·001 tonne.
1 tonne = 1·016 tons.

1 grain = 0·0648 gram.
1 ounce = 28·35 grams.
1 lb. (16 oz. or 7000 grains) = 453·6 grams.

TABLE II. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

TABLE II. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE III. ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

TABLE III. ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE IV.

Angle. Deg.	Angle. Radians.	Chords.	Sine.	Tangent.	Cotangent.	Cosine.			
0	0	0	0	0	∞	1	1·414	1·5708	**90°**
1	·0175	·017	·0175	·0175	57·2900	[illegible]	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	[illegible]	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	[illegible]	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3006	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	[illegible]	1·351	1·4835	**85**
6	·1047	·105	·1045	·1051	9·5144	[illegible]	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	[illegible]	1·325	1·4486	83
8	·1396	·139	·1392	·1405	7·1154	[illegible]	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	**80**
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·246	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·217	1·3090	**75**
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	**70**
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	**65**
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	**60**
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	**55**
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	**50**
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	**45**
			Cosine.	Cotangent.	Tangent.	Sine.	Chords.	Angle. Radians.	Angle. Deg.

ANSWERS

Exercises I. (p. 2.)

1. 9327. **2.** 10656. **3.** 11309.
4. 128150. **5.** 137967. **6.** 676395.
7. 183217. **8.** 451367. **9.** 1082490.
10. 2491034. **11.** £659. 11*s.* 6*d.* **12.** £876. 5*s.* 7*d.*
13. £1043. 19*s.* 7*d.* **14.** £664. 8*s.* 8*d.* **15.** £293. 10*s.* 6*d.*

Exercises II. (p. 3.)

1. 596288. **2.** 8714584. **3.** 292002788.
4. 33582720. **5.** 65580864. **6.** 49936864.
7. 2*s.* 2*d.*; 13*s.* **8.** 14*s.* $0\frac{3}{4}d.$; £1. 10*s.* $11\frac{1}{4}d.$
9. £386. 2*s.* 9*d.* **10.** 10 tons 9 cwt. 71 lb.
11. 35 tons 3 cwt. 44 lb. **12.** £1681. 14*s.* 6*d.*
13. 6·781 miles. **14.** $1\frac{1}{2}d.$

Exercises III. (p. 4.)

1. 6514. **2.** 255892. **3.** 175561.
4. 557779. **5.** 82557. **6.** 78952.
7. 252117. **8.** 628314. **9.** £20. 4*s.* 11*d.*
10. £28. 17*s.* 10*d.* **11.** 8 tons 18 cwt.

Exercises IV. (p. 5.)

1. 9887. **2.** 28201. **3.** 39765.
4. 148647. **5.** 6692, rem. 55. **6.** 4576, rem. 4.
7. 57013, rem. 13. **8.** 36, rem. 44. **9.** 215.
10. 571. **11.** 81. **12.** 19.
13. 1*s.* $11\frac{3}{4}d.$ **14.** 10 h. 55 m. 1 sec. **15.** 15 tons 15 cwt. 1 qr. 16 lb.

Miscellaneous Exercises V. (p. 11.)

1. £442. 6*s*. 7½*d*. **2.** £207. 18*s*. 10½*d*. **3.** £1572. 18*s*.
4. £4. 10*s*. 3*d*. **5.** £179. 7*s*. 6*d*. **6.** £3. 18*s*. 9·4*d*.
7. 37. **8.** £3668. 15*s*. **9.** £8. 10*s*. 9*d*.
10. 1980. **11.** 37·26 lb. **12.** 4*s*. 8*d*.
13. $11\frac{5}{11}d.$, $254\frac{6}{11}$ lb. **14.** 2 ft. 3·48 in. **15.** $x+y$; 80.
16. $n+x$; 50. **17.** $b+4+x$; 31. **18.** £15. **19.** 250.
20. (i) $a^2\times a^3$, (ii) a^3b^2. **21.** (i) 4, (ii) 32, (iii) 1056. **22.** xy; $m=n$; 2
23. $4n+m$; 44. **24.** $x-5$. **25.** $5-x$.
26. $20-x$. **27.** $9b$. **28.** $x-9$.
29. 11. **30.** 7. **31.** 5. **32.** $3x-p$.
33. $5x^2-2x-p$. **34.** $5x^2-2x+p$. **35.** $2x^2-3x$.

Exercises VI. (p. 16.)

1. 86602·11. **2.** 396·294888. **3.** 433·50198.
4. 175·353442. **5.** 1822·140783. **6.** 345·26579.
7. 28·95305. **8.** 19·466099. **9.** 1070·52825.
10. 19208·081. **11.** 94·6196. **12.** 66·8224.
13. 662·983028. **14.** 800·911183. **15.** 200·9839166.
16. 6·21004. **17.** 46·46561. **18.** 29·634 in.

Exercises VII. (p. 19.)

2. 0·3 cm. **3.** 0·38 cm. **4.** 22·16 cm.
5. (i) 381, 558·8, 685·8, 1193·8; (ii) 39·9, 24·9, 18·8, 33·1.

Exercises VIII. (p. 22.)

1. 0·137. **2.** 26, rem. 0·33 cm. **3.** 3069, 935·4.
4. 35·56 cm. **5.** 1·9 miles.
6. (i) 0·0254, (ii) 0·3048, (iii) 0·914, (iv) 1609·33.
7. (i) 32·61, (ii) 351, (iii) 168·98. **8.** 8 m. 7 dcm. 5 cm.
9. 1½. **10.** 100. **11.** 7½. **12.** 2½. **13.** 10000.

Exercises IX. (p. 27)

1. $\frac{5}{6}$. **2.** $\frac{47}{60}$. **3.** $\frac{73}{168}$. **4.** $2\frac{1}{4}$.

5. $14\frac{1}{5}$. **6.** $6\frac{2}{15}$. **7.** $8\frac{1}{3}$. **8.** $2\frac{6}{7}$.

9. $12\frac{1}{14}$. **10.** 14. **11.** $15\frac{1}{3}$. **12.** $\frac{11a}{6x}$.

13. $17\frac{5}{6}$. **14.** $3\frac{37}{55}$. **15.** $5\frac{67}{91}$. **16.** $1\frac{19}{52}$.

17. $3\frac{2}{3}$. **18.** $\frac{5}{6}$. **19.** $2\frac{197}{378}$. **20.** $\frac{1-3c}{6ac}$.

21. $17\frac{89}{252}$ yds. **22.** 3 tons 2 cwt. $42\frac{6}{11}$ lb. **23.** £1.

24. $\frac{2}{a-b}$, $\frac{2}{3}$. **25.** $\frac{2a+1}{15}$, $\frac{11}{15}$. **26.** $\frac{27x+44y}{60}$, $\frac{25}{12}$. **27.** $\frac{3}{4}(x-y)$, $\frac{3}{2}$.

28. $\frac{5a}{4x^2y}$, $\frac{5}{9}$. **29.** $\frac{a^2+ax+x^2}{a+x}$, $\frac{37}{7}$. **30.** $\frac{1}{1+2x}$, $\frac{1}{5}$.

31. $\frac{3x+7}{x^2+5x+6}$. **32.** $\frac{3x-7}{x^2-5x+6}$. **33.** $\frac{11b^2}{7ax^2}$, $\frac{11}{63}$.

34. $\frac{a^3b^2}{\sqrt{c}}$, 79·97. **36.** $\frac{x^4\sqrt{y}}{z^3}$, 3·073.

Exercises X. (p. 36).

1. 67·37°, 22·63°, 90°. **2.** 53·2°, 59·4°, 67·4°.

3. 36·8°, 93·2°, 9·7 ft. **4.** 31·38°, 108·62°, 8·9 cm.

5. 70·5°, 59°, 50·5°. **6.** 55·76°, 41·4°, 82·84°.

Exercises XI. (p. 45).

1. $\frac{3}{8}$. **2.** $\frac{21}{64}$. **3.** $\frac{25}{224}$. **4.** $\frac{5}{1024}$.

5. 3. **6.** 1. **7.** $\frac{9}{16}$. **8.** 1.

9. 18. **10.** $\frac{2}{3}$. **11.** $3\frac{1}{16}$. **12.** $\frac{3}{5}$.

13. $\frac{4}{3}$. **14.** $\frac{4}{5}$. **15.** $\frac{5}{3}$. **16.** $3\frac{3}{4}$.

17. $\frac{7}{40}$. **18.** $2\frac{5}{14}$. **19.** $\frac{1}{32}$. **20.** $4\frac{1}{2}$.

21. 3. **22.** $\frac{1}{4}$. **23.** $\frac{61}{495}$, $\frac{1}{7}$, $\frac{145}{444}$.

24. 1. **25.** $\frac{x^2}{8y^2}$. **26.** $\frac{x+3}{x+4}$. **27.** $9ab^4c^2$.

Miscellaneous Exercises XII. (p. 47).

1. $3\frac{9}{17}$. **2.** $1\frac{23}{60}$. **3.** 73. **4.** $\frac{7}{8}$.
5. $13\frac{3}{5}$. **6.** $\frac{17}{833}$. **7.** $3\frac{3}{28}$. **8.** $25\frac{3}{4}$.
9. $\frac{10}{33}$. **10.** $\frac{3}{7}$. **11.** 11 acres.
12. 13*s.* 8*d.*, £85. 8*s.* 4*d.* **13.** £40, £24, £16.
14. 56 min. 15 sec. **15.** $1\frac{1}{2}$ hrs. **16.** $2\frac{1}{2}$ min.

Exercises XIII. (p. 49).

1. 836·113. **2.** The latter, 0·03. **3.** 5574. **4.** £84.
5. 2368. **6.** 20·4. **7.** 38·75. **8.** 2516.

Exercises XIV. (p. 53.)

1. 4895·472. **2.** 34176·6324. **3.** 978658·0537.
4. 274163·278. **5.** 19·04377. **6.** 0·147637.
7. 210·95506. **8.** 0·054. **9.** 13·19.
10. 24·495. **11.** 33·51. **12.** 166·648.
13. 3565·345. **14.** 99·99. **15.** 0·0364.
16. 1·258. **17.** 125·449. **18.** 3·124.
19. 284·7. **20.** (i) 62·25 sq. ft., (ii) 159·69 sq. ft.
21. 330·06 sq. in. **22.** 3499·4.
23. 64·403 in. **24.** 76·18 in., 461·86 sq. in.

Exercises XV. (p. 55.)

1. 130·22. **2.** 776·4. **3.** 87631·5.
4. 4787·6. **5.** 372·547. **6.** 0·35.
7. 0·00698. **8.** 0·00598. **9.** 0·031.
10. 36·289. **11.** 0·005763. **12.** 8·075.
13. 3·00091. **14.** 14·00014. **15.** 34·00327.
16. 7·06. **17.** 0·00035. **18.** 0·51.

Miscellaneous Exercises XVI. (p. 58.)

1. 0·03068. **2.** 0·045, 0·035, 0·0002, 8, 8·0802.
3. 66. **4.** £2. 6*s*. 10·6*d*.
5. 0·25, 0·64, 0·86, 3·5, 0·03. **6.** 0·1859. **7.** $\frac{11}{9}$.
8. 4·9198. **9.** 8·2486. **10.** 0·03675.
11. $6\frac{7}{11}$. **12.** (*a*) 0·1563, (*b*) $4\frac{1}{2}d$.
13. (*a*) 0·7688, (*b*) 28 lb. **14.** (*a*) 0·275, (*b*) 9 cwt. 3 qr.
15. (i) $\frac{41}{180}$, (ii) 13*s*. $2\frac{2}{5}d$. **16.** (i) £3. 17*s*. 9*d*.; (ii) $\frac{1}{3}$.
17. 600, 350, 1050. **18.** £1700, £2550, £4250. **19.** $7\frac{1}{6}$. **20.** $\frac{1}{2}\frac{7}{0}$.

Exercises XVII. (p. 61.)

1. £2. 15*s*. **2.** $12\frac{5}{6}$ miles. **3.** 30 ft. 10 in.
4. 30·04 in. **5.** 2133. **6.** 37·45 miles, 7·8 min.
7. 39 miles per hour. **8.** 2 ft. 11 in. **9.** 10 st. 4·9 lb., 7 st. 2·8 lb.
10. 12 st. 2 lb. **11.** 4. **12.** $\frac{22s}{15x}$, 44 ft.

Exercises XVIII. (p. 66.)

6. 11, 220. **7.** 56·8. **8.** 49·4. **9.** 1·76.
10. 257·8. **11.** 201·3. **12.** 0·2775. **13.** 1·892.

Exercises XIX. (p. 74.)

1. 25. **2.** 5243. **3.** 9·345, 28·567.
4. 7·25. **5.** 91·5416. **6.** 4321.
7. 527. **8.** 409. **9.** 1079.
10. 257. **11.** 576. **12.** 10·588.
13. 1·732. **14.** 4·875. **15.** 165 yds. **16.** 5·6 ft.

Exercises XX. (p. 78.)

1. 5·06. **2.** 21·56. **3.** 1·54.
4. (i) 21·34, (ii) 4·4. **5.** (i) 7·72, (ii) 5·79. **6.** (i) 8·27, (ii) 9·25.
7. 2500. **8.** 19·62. **9.** 273·9.
10. 0·97. **11.** 191·6. **12.** 1226 grains.
13. 54432. **14.** (i) 1543·2, (ii) 220·36. **15.** 1587·26, 721·4.

Exercises XXI. (p. 81.)

2. 120 miles. **3.** £1176, £1960, £2744.
4. £13600, £8500, £5100, £1700. **5.** £4200, £3000, £1800, £1200.
6. £963. 3*s.* 3*d.*, £1284. 4*s.* 4*d.*, £2247. 7*s.* 7*d.*, £3531. 11*s.* 11*d.*
7. 17 : 28. **8.** 107·2 lb., 32·1 lb., 10·7 lb.
9. 76 oz. copper, 3·2 oz. tin, 0·8 oz. zinc. **10.** 50 lb.
11. Nitre 170·24 lb., sulphur 21·28 lb., charcoal 32·48 lb.
12. 43750 francs, 1 to 31. **13.** 2380 lb. **14.** 73·47.

Exercises XXII. (p. 83.)

1. 6. **2.** 900 gms. **3.** 0·786. **4.** 4·167.
5. 0·946. **6.** 1·26. **7.** 450 lb. **8.** 0·72.
9. $3\frac{3}{7}$ lb. **10.** Brass 8·1, glycerine 1·2. **11.** 7·1.
12. (i) 19·23, (ii) 1·85, (iii) 1·23, (iv) 0·81.

Exercises XXIII. (p. 91.)

1. 32 **2.** $2\frac{1}{4}$. **3.** 216 miles.
4. 12 men. **5.** $4\frac{1}{20}$ days.
6. 10240, £12. 16*s.*; 8063, £10. 1*s.* 7·2*d.* **7.** 11 weeks.

Exercises XXIV. (p. 94.)

1. 9·6 in. **2.** 72·8 lb. copper, 39·2 lb. zinc.
3. 248 lb. copper, 148 lb. zinc, 4 lb. tin.
4. 75 %, 10 %, 15 %; 84 lb. nitre, 11·2 lb. sulphur, 16·8 lb. charcoal.
5. 25 per cent. **6.** $7\frac{1}{2}$ per cent. **7.** $8\frac{1}{3}$ per cent.
8. £224, £240, £350; 27·52 %, 29·48 %, 43 %.
9. Nitre 304 lb., sulphur 38 lb., charcoal 58 lb.
10. £7, £11. 13*s.* 4*d.*, £16. 16*s.* 8*d.*, £21; 12·5 %, 20·8 %, 29·2 %, 37·5 %.
11. £640, £720, £768, £800; 21·7 %, 24·6 %, 26·3 %, 27·4 %.
12. 29·84 lb. **13.** 78·39 grains. **14.** 653 lb.
15. Carbon 78·86 %, hydrogen 6·42 %, oxygen 5·862 %, nitrogen 3·908 %, ash 4·95 %.
16. 0·000355 cub. in. **17.** 3400. **18.** 4·48 lb. **19.** 2195 lb.

Exercises XXV. (p. 96.)

1. £29. 12*s*. **2.** £27. 13*s*. 6½*d*. **3.** £391. 4*s*. 5¼*d*.
4. £64. 17*s*. 5·4*d*. **5.** £1. 8*s*. 6*d*. **6.** £883. 3*s*. 3*d*.
7. £12. 7*s*. 4¼*d*. **8.** £3215. 16*s*. 8*d*.
9. £457. 12*s*. 6*d*. **10.** £2. 4*s*. 11*d*.

Exercises XXVI. (p. 105.)

1. 1·8751. **2.** 1·5563. **3.** 1·9956. **4.** 0·8751.
5. 0·5563. **6.** $\bar{3}$·8751. **7.** $\bar{4}$·5563. **8.** 2·8785.
9. $\bar{2}$·8785. **10.** $\bar{5}$·8785. **11.** 1·4852. **12.** 0·6133.
13. (i) 0·3630, (ii) $\bar{3}$·3630, (iii) $\bar{5}$·3630.
14. (i) 1·5710, (ii) 0·5710, (iii) $\bar{3}$·5710.
15. 714·1. **16.** 1·176. **17.** 48·55. **18.** 0·01561.
19. 0·00917. **20.** 2·396. **21.** 0·0006398. **22.** 0·3607.
23. 1265. **24.** 27070. **25.** 0·0004511.

Exercises XXVII. (p. 107.)

1. 1065. **2.** 72·46. **3.** 0·07993. **4.** 287·4.
5. 984·2. **6.** 6·474. **7.** 750·2. **8.** 0·1233.
9. 148·9. **10.** 21·82.
11. (i) 125·4, (ii) 3·123, (iii) 50·34, (iv) 284·7, (v) 31·06.

Exercises XXVIII. (p. 108.)

1. 0·4006. **2.** 0·04369. **3.** 53·31.
4. 0·4997. **5.** 2·749. **6.** 58·76.
7. 7·658. **8.** 3·418, 0·3002. **9.** 0·1719, 1100.
10. 0·03968, 7501. **11.** 70·69, 3·736. **12.** 0·00325, 76·93.
13. 1704. **14.** 2817. **15.** 3·426.
16. 0·00007501. **17.** 0·3015.

Exercises XXIX. (p. 111.)

1. 2·06. **2.** 3·339. **3.** 1·93. **4.** 1·558.
5. 1·337. **6.** 1·81. **7.** 3·131. **8.** 1·36.
9. 13·32. **10.** 0·01159. **11.** 0·02825, 0·9649.
12. 5·046. **13.** 1·078. **14.** $3{\cdot}391 \times 10^7$. **15.** 1·703.
16. 2390. **17.** 0·5383. **18.** 16·25. **19.** 11·62.
20. 47·92. **21.** 0·05133. **22.** 1·673. **23.** 7·638.
24. 7·731. **25.** 398·9.

Exercises XXX. (p. 120.)

1. £160. 3*s.* 0*d.* **2.** £129. 17*s.* $4\frac{3}{4}d.$ **3.** £60. 15*s.*
4. £48. 9*s.* $4\frac{1}{4}d.$ **5.** 10 years. **6.** $4\frac{1}{2}$ per cent.
7. $4\frac{1}{2}$ years. **8.** £375. **9.** 4 per cent.
10. £4200. 10*s.* **11.** $3\frac{1}{3}$ per cent. **12.** £1739. 2*s.* 7*d.*
13. £1320. **14.** £187. 10*s.* **15.** £314. 11*s.* 8*d.*
16. 19*s.* 3*d.* **17.** £350. **18.** £1500.
19. £402. 10*s.*, £405. 16*s.* **20.** £2054. 1*s.* 11*d.* **21.** 16·45 years.
22. £224. 18*s.* **23.** £306. 16*s.* **24.** £1328. 10*s.*
25. £2271. **26.** £117. 14*s.* **28.** £2. 15*s.* **29.** £4. 10*s.*

Exercises XXXI. (p. 124.)

1. 150. **2.** The former. **3.** £1470.
4. £1856. 3*s.* $8\frac{1}{2}d.$, £65. 7*s.* 7*d.* **5.** £3000.
6. 6 per cents.; 13*s.* 4*d.* **7.** $7\frac{1}{2}$ per cent. **8.** Loss £8. 4*s.*
9. $3\frac{1}{33}$, £34590. **10.** 3·35, £50. 5*s.* 0*d.* **11.** £7680, £211. 4*s.*
12. $3\frac{1}{5}$ per cent. **13.** Decrease £1. 9*s.* 3*d.*
14. $2\frac{6}{7}$ years. **15.** £6. 14*s.* 6*d.*, £3. 19*s.* 6*d.*

Exercises XXXII. (p. 132.)

1. 242 yds. **2.** £33. **3.** $7\frac{1}{2}$ metres.
4. £5. 10*s.* **5.** 110 yds, £33. 11*s.* 9·6*d.* **6.** 35·49 ft.
7. 1186 francs. **8.** 21150, £26. 8*s.* 9·6*d.* **9.** £13. 8*s.* 8*d.*
10. 2·11, 6·33 ft.; 13·36 sq. ft. **11.** 250. **12.** £69. 1*s.*
13. 35·89, 36·01, $\frac{1}{8}$ %. **14.** 464·1 francs.
15. 3·2 ac. **16.** 0·000014. **17.** £1. 1*s.*

Exercises XXXIII. (p. 136.)

1. 82·5 sq. ft. **2.** 3·5 in. **3.** 9·397 ft.
4. 9 ft. 11·78 in. **5.** 625·3 sq. ft. **6.** 18·8 ac.
7. 92·4 sq. in., 14·94 in. **8.** 20·97. **9.** 756 sq. ft.
10. 0·07875 sq. m. **11.** £30. **12.** 1·464 ac.
13. £42. 0*s.* 4·8*d.* **14.** 10 ft. **15.** 3·968 ac.
16. 6·548 ac. **17.** 776·45 sq. ft.

Exercises XXXIV. (p. 139.)

1. 92 ft. **2.** 17835 sq. miles. **4.** 4 ft.
5. 408·54 sq. ft. **6.** 8, 12 ft. **7.** 400 sq. ft.
8. 1350 sq. yds. **9.** 250 yds. **10.** 30 ft.

Exercises XXXV. (p. 146.)

1. (i) 26·08, (ii) 30·48, (iii) 171·6, (iv) 240·1.
2. (i) 71, (ii) 31·1, (iii) 32·8, (iv) 42·5. **3.** 632 yds. **4.** 51·2.
5. (i) 24859 miles, (ii) 24856 miles. **6.** £5. **7.** 10·61 ft.
8. 3·978 ft., 3379. **9.** 4·5 in. **10.** 96·26 in.

Exercises XXXVI. (p. 151.)

1. 4·792 sq. in. **2.** 8·296 sq. in.
3. (i) 0·008628, (ii) 0·0002138, (iii) 0·0004908 sq. in.
4. (i) 0·1482, (ii) 0·02333, (iii) 0·03535 in.
5. (i) 0·05146, (ii) 0·0009241 sq. in., (iii) 0·00407 sq. in.
6. (i) 0·181, (ii) 0·02425, (iii) 0·05091.
7. 4792 sq. yds., 31·42 yds. **8.** 18·06 sq. in.
9. 11·34 sq. in. **10.** 75·41 sq. in. **11.** 37·7 sq. in.
13. 0·3183 sq. ft. **14.** 706·8 sq. in. **15.** 1963 lb.
16. 10·19. **17.** 9·473 sq. m.
18. 1·762 ft. **19.** 7·429 ft., 4·388 sq. ft.

Exercises XXXVII. (p. 155.)

1. 1800 sq. ft. **2.** 22288 sq. ft. **3.** 336·7 sq. ft.
4. 674·08 sq. ft. **5.** 424·8 sq. ft. **6.** 443·1 sq. yds.
7. 607·8 sq. in. **8.** 180·5 lb. **9.** 10·62 sq. in.

Exercises XXXVIII. (p. 159.)

1. (i) 1·57, (ii) 1·96, (iii) 6·28, (iv) 31·4, (v) 40·1.
2. (i) 3·14, (ii) 4·9, (iii) 11, (iv) 15·9, (v) 60·1 sq. in.
3. (i) $\frac{5}{8}$, (ii) $1\frac{3}{4}$, (iii) $4\frac{5}{8}$, (iv) 9·5 sq. in.
4. (i) 1·5, (ii) $2\frac{1}{4}$, (iii) $5\frac{3}{4}$, (iv) 6·3 in.

Exercises XXXIX. (p. 168.)

1. 337·8 sq. ft., 519·4 sq. ft. **2.** 24·74 sq. ft., 43·98 sq. ft.
3. 10·61 ft. **4.** 58·24 sq. ft., 74·24 sq. ft.
5. 58·31 sq. ft. **6.** 12·16 ft. **7.** 63420 sq. cm.
8. 11 ft. **9.** 63·23 sq. metres. **10.** 81·74 sq. cm.
11. 9·77 cm. **12.** 1·345 sq. ft. **13.** 7 in.
14. 8015 sq. ft., $12\frac{5}{8}$ ft. **15.** 4 in. **16.** 12·74 ft., 11·7 ft.

Exercises XL. (p. 170.)

1. 1764 lb. **2.** 0·125 in., 0·1014 in., 0·084 in.
3. 1·102, 0·527, 0·12 in. **4.** 9497 kilog., 1·442 m.
5. 4·478 tons. **6.** 365·8 lb. **7.** 8·845 ft.
8. 1776 lb., 12·66 tons. **9.** 2 in. **10.** 1 ft. **11.** 8 ft.
12. 422 cub. ft., 30 tons. **13.** 7982 cub. in. **14.** 2·08 lb.
15. 45 : 12, 25 : 4, 9000 lb., 40 lb. **16.** 200. **17.** 3198 lb.
18. 218·7 lb. **19.** 136·7 lb. **20.** 7140 litres, 13209 kilog.
21. 7 ft. 6 in. **22.** $\frac{1}{1\frac{1}{2}}$ in. **23.** 6497.
24. 822200. **25.** 38·28 cub. ft., 8·269 tons.
26. 5848 lb. **27.** 10470 sq. ft.
28. 3·117 ft., 6·873 ft. **29.** 1092·6 sq. ft., 6·365 tons.
30. 100 galls. **31.** $37\frac{1}{3}$. **32.** 14 cub. ft., £1. 8*s*.
33. 60000 cub. ft. **34.** 2000 cub. in.

Exercises XLI. (p. 175.)

1. 450·9 lb. **2.** 247·7 lb. **3.** 5248 lb.
4. 98·02 lb. **5.** 246 lb., 3 in. **6.** 1555·5 lb.
7. 1·857 cub. ft., 930·5 lb. **8.** 2557·8 cub. in., 741·8 lb.
9. 0·34, 0·67, 1·0, 1·68, 3·36 in. **10.** 9·29 grams.
11. 8·17, 0·1748 cm. **12.** 5·13 kilog. **13.** 0·1203 cm.
14. 13·81 sq. ft., 22·04 lb. **15.** 186·7 lb. **16.** 475 kilog., 1045 lb.
17. 377 sq. in., 603·2 sq. in., 1131 cub. in., 337 lb.
18. 237·9 lb. **19.** 2 ft. **20.** 4799. **21.** 14*s*. **22.** £15720.
23. 1352 cub. in., 351·7 lb. **24.** 1 ft. **25.** 4·676 lb.
26. 201·7 lb. **27.** 21·65 cub. ft., 9727 lb. **28.** 0·87.
29. 50 lb. **30.** 9·9 ft. **31.** 4434 lb.
32. £81. 16*s*. 2*d*., £35. 7*s*. **33.** 73·82 metres. **34.** £1. 17*s*. 8*d*.
35. 217·2 yds., 7·7 lb. **36.** 63·65 ft. **37.** 134 tons. **39.** 0·37 cm.

Exercises XLII. (p. 184.)

1. 96 cub. in. **2.** 113·31 sq. ft., 64 cub. ft.
3. 128·3 cub. in., 33·36 lb., 53·1 lb. **4.** 4 in.
5. 7·966 in. **6.** 532 cub. ft. **7.** 1·31 cub. in.
8. 56·56 cub. ft. **9.** 46·77 cub. ft. **10.** 9 ft.

Exercises XLIII. (p. 189.)

1. 179·6 cub. in., 46·7 lb. **2.** 89·39 lb.
3. 655·4 lb. **4.** 18·87 lb., 5·2 in. **5.** 4·044 in.
6. (i) 6·263 lb.; (ii) 3·183 lb.; (iii) 6·415 lb., 2·4 %, 3·207 lb., 0·75 %.
7. 1 gall. **8.** 8·9 lb. **9.** 20 in.
10. 364674 grams. **11.** 11·6 in. **12.** 12·62 in.
13. 1·46 cub. in. **14.** 139·9 cub. in., 36·37 lb.
15. 2385. **16.** 0·5376 grams.

Miscellaneous Exercises XLIV. (p. 190.)

1. £8. 5*s*. 3·8*d*. **2.** 2·07 sq. ft.
3. 39·24 yds., 259·1 sq. yds. **4.** 1111 sq. ft.
5. 227·5 sq. in. **6.** 5 ft. 4 in.
7. 23·03 ft. **8.** 1·274 ft.
9. 11·46, 9, 6·93 sq. in. **10.** 690 m., 2·07 min.
11. 72·636, 74·292, mean 73·464 cub. in., 1·27 .. **12.** 260·6 sq. in.
13. (i) 615·8 sq. ft., (ii) 756·7 sq. ft., (iii) 2463 sq. ft.
14. (i) 82·5 sq. ft., (ii) 756 sq. ft., (iii) 0·3183 sq. ft., (iv) 62·23 sq. ft.
15. 16·57 lb. **16.** 83100. **17.** $\frac{1}{2}$ in.
18. $53\frac{1}{3}$ ft. **19.** 0·01953 in. **20.** 275·4 lb.
21. 344 sq. ft., 3·58 cub. ft. **22.** 13021 c.c.
23. £1. 2*s*. 6*d*. **24.** 90·74 tons, £21. 16*s*.
25. 150·8 sq. in., 2·727 tons. **26.** 83·79 cub. ft., 2059 lb.
27. 5·819 cub. ft. **28.** 43 sq. ft., 20·37 cub. ft.
29. 16 in. **30.** 3850 ft. per min.
31. 776·45 sq. ft. **32.** 922·3.
33. 143·3 sq. ft., £10. 15*s*. **34.** 13·68 tons.
35. 5271 cub. ft., 3574 cub. ft. **36.** 7974·4 lb.
37. 11·39 lb. **38.** (i) 11490 cub. ft., (ii) 2815 cub. ft.
39. 32980 cub. ft., 4·86 ft. per sec. **40.** 31·11 lb.
41. 11·5 in., 58·35 sq. in., 2·276 lb. **42.** 87·45 cub. in., 26·25 lb.
43. 1·25 in. **44.** 6 in.
45. 329·4 sq. in., 563·7 cub. in., 146·6 lb.
46. 1 in. **47.** 6·25 in. **48.** 7·2 in. **49.** 6·9 in.

Exercises XLV. (p. 199.)

1. 0·41, 0·91. **2.** 0·81, 0·59. **3.** 23°, 0·92, 0·42.
4. 36°, 0·59, 0·73. **5.** 42°, 0·67, 0·74. **6.** 35°, 0·57, 0·82.
7. 43·4°, 3·169 in., 4·36 in. **8.** 2·151 in., 4·088 sq. in., 5·11 in.
9. 7·185 sq. in., 5·388 sq. in., $\frac{4}{3}$.
10. $CA = 8{\cdot}264$ in., $B = 55°\ 44'$, $\sin B = 0{\cdot}8264$, $\cos B = 0{\cdot}563$, $\tan B = 1{\cdot}4677$.

13. 0·5348, 0·845. **15.** 0·8601, 1·686.

16. 13·89 in. **17.** $OP = 6{\cdot}49$ in., $XOP = 35° 58'$.

18. 40°, 6·428 in., 6·84 in., 6·981 in., $\sin B = 0{\cdot}684$, $B = 0{\cdot}6981$ radians.

19. 2·309 in. **20.** 2·11 cm.

21. 2·33 in., $B = 43° 30'$, $\sin B = 0{\cdot}6884$, 4·212 sq. in., 4·176 sq. in., mean 4·194 sq. in., 0·42 per cent.

22. 3·7 in., 0·8 in., 2·6 in., 6·29 sq. in., 3·7 in., 65°, 6·2 sq. in., mean 6·245 sq. in., 0·72 per cent.

Exercises XLVI. (p. 202.)

1. 62, 46. **2.** 54, 320. **3.** 20, 40.

4. 418 days 8 hrs. 32 min. **5.** 400 lb. **6.** 34·6.

7. 21 min. 36 sec. **8.** 17*s.* 6·2*d.*, 15*s.* 2·7*d.* **9.** 4, 19136.

10. £16. 4*s.* 0½*d.* **11.** 166·716. **12.** 28706, 47838.

13. £2. 4*s.* 11*d.* **14.** $\frac{3}{8}$ gall. **15.** $3\frac{7}{15}$ oz

16. 48 ft. **17.** 5000. **18.** 54 lb.

19. 492·8 lb. **20.** (i) 6·25 cm., (ii) 4·9 m., (iii) 64 cm.

21. 87·96 days. **22.** 10, 4. **23.** (i) 6×10^{27}, (ii) 6×10^{21}.

24. 10·8 oz. **25.** (i) 3*s.*, 2*s.* 6*d.*, 10*d.*, (ii) 3*s.* 6*d.*, 12*s.* 6*d.*, 9*d.*, 3·15*d.*

26. $27\frac{3}{4}$ miles. **27.** 88·9 per cent.

28. 15·38 per cent., 11·1 per cent. **29.** 62·5 per cent.

30. 9%, 50%. **31.** 56%. **32.** 15·5.

33. 300%. **34.** 333·9 sq. cm. **35.** 6642 lb.

36. 121 yds., 402 yds. **37.** 1080 sq. ft. **38.** $24\frac{3}{4}$ ft.

39. $8{\cdot}612 \times 10^6$. **40.** 1040, 282·45 ft. **41.** 3·15 c.c.

42. 8·68 c.c. **43.** 0·53, 33 lb. **44.** 8320·8 cub. in., 1·1 tons.

45. 2484. **46.** 1000 yds. **47.** 154 tons.

48. 306 cub. ft., 15 tons. **49.** 4352.

50. 48·5 cub. ft., $2\frac{1}{4}$ tons. **51.** 3150. **52.** 30780 cub. yds.

53. 1·571 sq. ft. **54.** 11 ft. **55.** 1187 c.c., 10·08 lb.

56. 52·36 sq. in. **57.** 384·5 grams. **58.** 6·2 mm.

59. 13·5 cm. **60.** 22·5 lb. **61.** 133·5 gms.

62. 2156·6 lb. **63.** 127 cub. in. **64.** ½ in. **65.** 657·9 lb.

Exercises XLVII. (p. 218).

1. $1\frac{5}{8}$ in., $2\frac{3}{8}$ in., $3\frac{5}{8}$ in. **2.** 1·9 in. **3.** 294·9 ft.

4. 3·525 sec. **5.** 15·85. **6.** (i) 0·007492, (ii) 0·1199.

7. $7\frac{1}{2}$ in., 1·57 in. **8.** $42\frac{1}{3}$. **9.** (i) 7·34 ft., (ii) 29·36 ft.

10. (i) 0·78 in., (ii) 80·46. **11.** $253\frac{1}{3}$.

12. 4581·5. **13.** 195·8. **14.** 5237.

15. (i) 0·8486 in., (ii) 1·04 in., (iii) 1·123 in. **16.** 14·65.

17. 4·356. **18.** 3·67. **19.** 3·367 in.

20. 16·52. **21.** 4·511 ft. **22.** 0·6057 in.

23. 527·4 lb., 0·128. **24.** 451 lb. **25.** 0·025 amp.

26. 10 volts. **27.** 0·15 amp. **28.** 1 ohm.

29. 4 ohms. **30.** 3 amp. **31.** 30 ohms.

32. 50. **33.** $7\frac{1}{2}$ ohms. **34.** $C=\dfrac{10\times 1\cdot 07}{10\cdot 10-10\times 14-220}=0\cdot 023.$

35. 2·4 ohms. **36.** $e=E-CR=220-15\times 5=125$ volts.

37. (i) 3512, (ii) 32660, (iii) 43880.

38. (i) 8·28 miles, (ii) 22 miles per hour, (iii) 37·5 min.

39. (i) 15·07, (ii) 31·97. **40.** 585·9 lb.

41. 0·1067 in. **42.** $W=2806$ lb., $D=0\cdot 05716$. in.

43. (i) 0·51, (ii) 0·93, (iii) 2·36, (iv) 2·65. **44.** 92·92, 525·8.

45. (i) 27610 in lb., (ii) 2863000 in lb. **46.** 43·81.

INDEX.

Acute angled triangle, 34.
Addition, 1; areas, 130; complementary, 4, 10; decimals, 15; generalised, 6; vulgar fractions, 23.
Addition and subtraction, of areas, 130.
Annulus, area of, 149.
Angles, measurement of, 29; plotting, 30.
Angular measurement, 195.
Arch, 143.
Arithmetical evaluation, 209.
Area of, circle, 147; ellipse, 149; equilateral triangle, 134; hexagon, 135; irregular figure, 152; measurement, 37; metric measures, 47; parallelogram, 127; polygon, 136; rectangle, 39, 126; trapezium, 137; triangle, 133, 134; quadrilateral, 138.
Averages, 59.

Brackets, 11.
British units, 13.
British weights, 76.
Brokerage, 122.

Calculation of logarithms, 112.
Carpeting, 129.
Characteristic, 100, 102.
Chords, scale of, 31.
Circle, area of, 147; circumference of, 140; to pass through three points, 143.
Circumference of, a circle, 140; an ellipse, 145.
Comparison of fractions, 24.
Complementary addition, 4, 10.
Compound interest, 113, 115.
Cone, 161; frustum of, 183; surface of, 165; volume of, 181.
Contracted multiplication, 51.
Conversion of fractions, 44.
Cross-section, 174.
Cube, 67; surface of, 162; volume of, 63.
Curved line, length of, 141.
Cylinder, 160; circumscribing, 167; surface of, 162.

Debenture stock, 122.
Decagon, 36.
Decimals, 14; addition of, 15; circulating or recurring, 45; concrete quantities, 56; division of, 54; multiplication of, 15, 49; subtraction of, 15.
Development, of a cube, 162; of a pyramid, 164.
Discount, and present worth, 118.
Division, 4; of decimals, 54; of fractions, 43; logarithms, 107.

Ellipse, 144; area of, 149; circumference of, 145.
Equilateral triangle, 34; area of, 134.
Evolution, 68; by logarithms, 110.

Figures, significant, 50.
Fractions, conversion of, 44; decimal, 14; division of, 43; multiplication of, 41; square root of, 72; vulgar, 23.
Fractional index, 98.
Frustum, of cone, 183; of pyramid, 182.

Graphs, 2.

Heptagon, 36.
Hexagon, 36; area of, 135.
Hollow circle, area of, 149.

Improper fractions, 24.
Indices, 97.
Index, 67; fractional, 98.
Interest, 95; compound, 95; simple, 95.
Inverse proportion, 86.
Involution, 67; by logarithms, 109.
Irregular figure, area of, 152.
Isosceles triangle, 34.
Italian method, 55.

Length, of curved line, 141; measures of, 13, 17.
Linings, 129.
Liquids, volume of, 64.
Logarithms, 100; calculation of, 112; division by, 107; evolution by, 110; involution by, 109; multiplication by, 106.
Logarithm of a number, 103.
Lowest common multiple, 26.

Mantissa, 100.
Mass, 75.
Mean proportional, 85.
Measurement, of angles, 29, 39, 195; of area, 37; of length, 13, 17, 224; metric, 17; of volume, 62, 169; of weight, 76, 77.
Metric measurement, area, 47; conversion, 19, 65, 77; length, 17; units, 13; weights, 77.
Mixed numbers, 25.
Multiplication, 2; contracted, 51; of decimals, 15, 49; of fractions, 41; generalised, 8; by logarithms, 106.

Negative sign, 7.
Nonagon, 36.

Obtuse angled triangle, 34.
Octagon, 36.

Parallelogram, 32; area of, 127.
Pentagon, 36.
Percentages, 91, 93.
Plotting of angles, 30.
Polygon, 36; area of, 136.
Prism, 160; rectangular, 63; surface of, 161; volume of, 63, 169.
Preference stock, 122.
Present worth, 118.
Protractors, 31.
Proportion, compound, 88; inverse, 86; simple, 88.
Pyramid, development of, 164; frustum of, 182; surface of, 163; volume of, 180.

Quadrilateral, 33; area of, 138.

Radian, 195.
Ratio, 78.
Ratios of an angle, 196; trigonometrical, 197.
Rectangle, 32; area of, 39, 126.
Recurring decimals, 45.
Rectangular beam, 209.
Rhombus, 32.
Right-angled triangle, 33.
Root, square, 69; of a fraction, 72; graphically, 72.
Rule, of signs, 9; Simpson's, 153; slide, 156.

Scale of chords, 31.
Shares and stocks, 121.
Significant figures, 50.
Simple interest, 113, 114; graphically, 117.

Simple and compound proportion, 88.
Simplification of fractional expressions, 46.
Simpson's Rule, 153.
Similar solids, 187.
Slide Rule, 156.
Specific gravity, 81.
Sphere, 166; surface of, 166; volume, 185.
Stocks, and shares, 121; debenture, 122; preference, 122.
Squared paper, 20, 38, 48, 73, 85, 89, 113, 117, 155, 184.
Square, 32; root, 6, 9.
Subtraction, 3; of areas, 130; of decimals, 15; generalised, 9; of vulgar fractions, 23.
Surface, of cube, 162; of cylinder, 162; of prism, 161; of pyramid, 163; of sphere, 166.

Triangle, 33; area of, 133; equilateral, 34; isosceles, 34.
Trapezium, 32; area of, 137.
Third proportional, 85.
Tetrahedron, 164.
Trigonometrical ratios, 197.
Tables of, Antilogarithms, 230; area, 224; capacity, 225; conversion, 226, 227; length, 224; logarithms, 228; metric, 225; money, 224; specific gravity, 83; time, 225; volume, 224; weights, 83, 225.

Unit angle, 195.
Units, British, 13; metric, 13.

Variation, 86.
Volume, of, annulus, 174; cone. 181; cube, 170; cylinder, 173; prism, 169; liquids, 64; sphere, 185.
Vulgar fractions, 23; comparison of, 24.

Weights, 75; British, 76; metric, 77.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.

WORKS BY FRANK CASTLE, M.I.M.E.

Globe 8vo. 6*s.*

A Manual of
Practical Mathematics

Royal College of Science Magazine.—"The most compact and comprehensive compendium of mathematics we know. . . . We have great pleasure in recommending this '*multum in parvo*' to all those of our readers who require to use mathematical methods for any purpose whatsoever."

School World.—"It is impossible in a short notice to do justice to a book that is so full of matter as this manual. . . . The manual is full of suggestiveness, and introduces the student, who has a fair acquaintance with mechanics and physics, to many of the most important applications of the higher mathematics to these subjects."

Globe 8vo. Price 2s. 6d. Key, 5s. net.

Practical Mathematics
for Beginners

Educational Times.—"Mr. Castle's little volume is likely to prove a very useful one."

Practical Teacher.—"We can cordially recommend the book, not only to those for whom it is intended, but to all teachers of mathematics who desire to make their lessons more real and interesting by introducing practical illustrations."

Globe 8vo. Price 3s. 6d.

Elementary
Practical Mathematics
for Technical Students

Secondary Education.—"The course embraces exactly what is requisite for students who desire to derive the full benefit of a course of instruction in technical work applicable to trades and industries. The book is admirably arranged. The diagrams are clear, and the examples numerous. . . . Every school teacher ought to have it as a guide to more practical methods in teaching."

School World.—"Sure to command a good sale. Fortunately it deserves this on its own merits. . . . Many of the subjects dealt with by the author might be introduced with great advantage into a school course; and his book deserves the careful attention of teachers from this point of view."

LONDON: MACMILLAN AND CO., LTD.

Works by FRANK CASTLE, M.I.M.E.

Globe 4to. Price 4s. 6d.

Machine Construction and Drawing

Nature—"The book can be cordially recommended for use in drawing classes and to young engineers who are seeking after knowledge on which to base subsequent work in machine design."

Teacher.—"An admirable publication produced in the firm's well-known excellent style. . . . We have nothing but high praise for the whole work, and strongly recommend it to intending students."

Globe 8vo. In Two Parts. 1s. 6d. each.

Workshop Mathematics

Nature—"They may fairly claim to provide far more inspiring information and serviceable exercises than can usually be found in text-books designed for use in schools."

Practical Engineer.—"We are sure that this work of the author is based upon the right lines, and will be most useful alike to teacher and student in their work."

Practical Teacher—"These books are admirably suited for classes of working-men, being the kind of 'Practical Mathematics' required in the workshop."

Globe 8vo. 2s.

Practical Arithmetic and Mensuration

School World.—"For pupils in continuation schools and in technical classes this seems to us to be an excellent text-book. The subject is presented in a thoroughly interesting manner."

Educational News.—"One of the most practical and up-to-date books on the market."

Mathematical Tables for Ready Reference.

Crown 8vo. Sewed. 2d.

Logarithmic and Other Tables for Schools.

8vo. Paper covers, 6d. Cloth limp, 8d.

Five-Figure Logarithmic and Other Tables.

Fcap. 4to. 1s.

LONDON: MACMILLAN AND CO., LTD.

·5236·

1·0472

·1047·

1·1519

~~22~~
~~22~~
~~44~~

484

22

968

968

10·648